Mathematical assessment of qualitative diagnostic calibration for respiratory inductive plethysmography

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De Groote, Anne, Manuel Paiva, and Yves Verbandt. Mathematical assessment of qualitative diagnostic calibration for respiratory inductive plethysmography. J Appl Physiol 90: 1025–1030, 2001.—We present a critical assessment of qualitative diagnostic calibration (QDC), which claims to provide a relative calibration of respiratory inductive plethysmography during natural breathing (Sackner MA, Watson H, Belzto AS, Feinerman D, Suarez M, Gonzalez G, Bizousky F, and Krieger B. J Appl Physiol 66: 410–420, 1989). QDC computes the calibration factor (K) by considering breaths of constant tidal volume (VT) and provides a criterion to select breaths when VT is unknown. We applied QDC on uncalibrated data constructed from simulated sets of thoracic and abdominal volumes, with a predefined K. As expected, QDC yields a correct K when applied to breaths at constant VT. In breathing at quasi-constant VT, the criterion for breath selection is shown to bias the results toward K = 1. For spontaneous breathing, the calculated K deviates from its predefined value and depends heavily on the selection criterion. We conclude that QDC will only provide a correct calibration factor when applied to an entire set of breaths with constant or quasi-constant VT. More generally, physiological conclusions based on QDC should be critically evaluated on a case-by-case basis.

QDC THEORY

We will briefly recall the theory underlying QDC (10). The two-compartment model of the respiratory system of Konno and Mead (6) expresses the change of volume measured at the mouth (∆Va) as the sum of a rib cage (∆Vrc) and an abdominal (∆Vab) contribution

$$\Delta V_a = \Delta V_{rc} + \Delta V_{ab}$$

These volume changes are obtained indirectly by measuring the variations of two representative thoracic and abdominal dimensions by inductive plethysmography, magnetometers, or strain gauges. Using the notations of Sackner et al. (10), Eq. 1 can be rewritten as

$$\Delta V_a = M(K \Delta u_{Vrc} + \Delta u_{Vab})$$

where the calibration factor K weights the relative contribution of the uncalibrated (u) electrical signals ∆uVrc and ∆uVab from the thoracic and abdominal compartments and M scales the sum (K ∆uVrc + ∆uVab) to ∆Va. The QDC as well as the isovolume calibration are methods yielding K. The isovolume calibration consists of having the subject shift volume forward use of standard gains (2) to the isovolume maneuver requiring active cooperation of the subject (6). From this practical point of view, QDC is very attractive because it is a single-posture method that is carried out during spontaneous breathing without need for a mouthpiece or a facemask. However, this method has been subjected to criticism: Sartene et al. (11) proposed a more rigorous formulation of the QDC coefficient, Thompson (13) concluded from mathematical considerations that QDC is not reliable, and Brown et al. (3) indicated that “a critical evaluation of the mathematics underlying the QDC method, including computer simulations of the potential errors that could arise under circumstances that violate the underlying assumptions, would be extremely valuable.”

In this paper, we analyze the principles and the mathematics underlying the QDC method, we test the approximations proposed by its authors to fulfill its implicit assumptions, and we evaluate its limits of application.

SINCE THE WORK OF Konno and Mead (6) modeling the chest wall movements with two degrees of freedom and introducing the isovolume maneuver, other methods have been proposed to calculate tidal volume (VT) from measurements of the movements of abdomen and rib cage. They are mostly based on a simultaneous recording of the breathing volume by a spirometer or a pneumotachograph (4, 5, 7, 9, 12). More recently, Sackner et al. (10) described the qualitative diagnostic calibration method (QDC), which is based on spontaneous breathing, and Banzett et al. (2) used standard calibration ratios to determine the appropriate gains for the rib cage and abdominal signals. These calibration methods differ in the information they provide: an absolute calibration against the volume (4, 5, 7, 9, 12) or a relative weighting of the thoracic and abdominal signals (2, 6, 10). Furthermore, the complexity of their implementation varies significantly from the straight-forward use of standard gains (2) to the isovolume maneuver requiring active cooperation of the subject (6). From this practical point of view, QDC is very attractive because it is a single-posture method that is carried out during spontaneous breathing without need for a mouthpiece or a facemask. However, this method has been subjected to criticism: Sartene et al. (11) proposed a more rigorous formulation of the QDC coefficient, Thompson (13) concluded from mathematical considerations that QDC is not reliable, and Brown et al. (3) indicated that “a critical evaluation of the mathematics underlying the QDC method, including computer simulations of the potential errors that could arise under circumstances that violate the underlying assumptions, would be extremely valuable.”

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1025
between the thoracic and abdominal compartments while the upper airways are occluded. Hence, \( \Delta V_a = 0 \) and Eq. 2 becomes

\[
K = -\Delta u_{Vab}/\Delta u_{Vrc}
\]

(3)

The QDC method (10) works at constant \( V_T \) instead of \( \Delta V_a = 0 \). It is based on a claimed observation that the breath-to-breath variations of \( \Delta V_{rc} \) and \( \Delta V_{ab} \) are normally distributed (10). With breathing at constant \( V_T \), calculating the standard deviation (SD) of each term of Eq. 2 and using \( SD(V_T) = 0 \) would yield according to Sackner et al. (10)

\[
K = -SD(\Delta u_{Vab})/SD(\Delta u_{Vrc})
\]

(4)

Because, during natural spontaneous respiration, breathing at constant \( V_T \) is not possible, Sackner et al. propose to approximate a constant \( V_T \) by collecting a large number of breaths and by excluding those whose sums show large deviations from the mean sum \( \mu(\Delta u_{Vrc} + \Delta u_{Vab}) \). The range to be considered is thus

\[
[\mu(\Delta u_{Vrc} + \Delta u_{Vab}) \pm x SD(\Delta u_{Vrc} + \Delta u_{Vab})]
\]

(5)

where \( x \) defines the interval width around the mean. According to Sackner et al., this chosen parameter should be between 0.6 and 1.

METHODS AND DATA

Our approach is based mainly on simulated data following Eq. 1 and consists of two parts. First, we analyze the mathematics underlying the QDC method. Second, we study in some detail the approximation of Eq. 5, which selects the breaths at constant \( V_T \). For this analysis, we start from simulated sets of thoracic and abdominal volumes and construct uncalibrated signals using a predefined value of \( K \).

The procedure is illustrated in Fig. 1: the sets of thoracic and abdominal volumes, \( \{ \Delta V_{rc} \} \) and \( \{ \Delta V_{ab} \} \), are obtained from a two-dimensional normal distribution, with major axis at angle \( \beta \) with respect to the abscissa, and centered around the mean \( \mu(\Delta V_{rc}, \Delta V_{ab}) \), with major axis at angle \( \beta \) with respect to the abscissa. The subscript \( i \) indicates the breath number in the data set. The sets are divided by two predefined calibration factors \( K_{rc} \) and \( K_{ab} \), which yield the uncalibrated thoracic and abdominal volumes \( \{ \Delta u_{Vrc} \} \) and \( \{ \Delta u_{Vab} \} \). In this operation, \( K_{rc}/K_{ab} \) represents the calibration value \( K \).

![Fig. 1. Construction of the simulated breathing data: the pairs of thoracic and abdominal volumes (\( \Delta V_{rc}, \Delta V_{ab} \)) are obtained from a two-dimensional normal distribution, centered around the mean (\( \mu(\Delta V_{rc}, \Delta V_{ab}) \), with major axis at angle \( \beta \) with respect to the abscissa. The subscript \( i \) indicates the breath number in the data set. The sets are divided by two predefined calibration factors \( K_{rc} \) and \( K_{ab} \), which yield the uncalibrated thoracic and abdominal volumes (\( \Delta u_{Vrc}, \Delta u_{Vab} \)). In this operation, \( K_{rc}/K_{ab} \) represents the calibration value \( K \).](https://example.com/figure1.png)

RESULTS

**Analysis of the mathematics of the QDC method.** It is shown in the APPENDIX that

\[
K = SD(\Delta u_{Vab})/SD(\Delta u_{Vrc})
\]

(6)

is actually the solution of Eq. 2 when the pairs (\( \Delta u_{Vrc} \), \( \Delta u_{Vab} \)) belong to a straight line. Note that there is no minus sign in Eq. 6. In this case, \( K \) is interpreted geometrically as the absolute value of the slope of the line defined by the set of uncalibrated volumes (\( \Delta u_{Vrc}, \Delta u_{Vab} \)). If the pairs (\( \Delta u_{Vrc}, \Delta u_{Vab} \)) do not belong to a straight line but are strongly correlated, \( K \) represents an approximation of the absolute value of the slope of the regression line calculated through the set (\( \Delta u_{Vrc}, \Delta u_{Vab} \)). This approximation is all the better when the points are strongly correlated.

**Approximation of Eq. 5 for simulated breathing at constant \( V_T \).** Figure 2A shows the distributions of thoracic, abdominal and volumetric data for the simulation, which corresponds to a respiration pattern at perfectly constant volume. The distribution of \( \{ V_T \} \) is thus an infinitesimally narrow peak, generally referred
to as a Dirac distribution. Note that the set of uncalibrated sum \( \Delta uV_{rc} + \Delta uV_{ab} \) does not follow a Dirac distribution. Figure 3A shows the calibrated and uncalibrated data in a Konno-Mead plot. Because \( V_T \) is constant, these data define two straight lines with slopes equal to \(-1\) and \(-0.7\), respectively (Eqs. 1 and 2). The selection of breaths according to \( x = 0.6 \) is shown in Fig. 3A by the two dotted lines of slope \(-1\). The proportion of breaths that are eliminated corresponds to \((1 - p)\) where \( p \) is the probability of being inside the interval \( (\mu - x \text{ SD}, \mu + x \text{ SD}) \). For \( x = 0.6 \), \( p \) equals 0.45. Hence, more than half of the total number of breaths (55%) are eliminated although they are, by construction, at constant \( V_T \). Nevertheless, the points that remain define always the same straight line with the slope of \(-0.7\). This slope thus provides a correct value for \( K \) in the particular case \( x = 0.6 \). Figure 4 shows \( K \) as a function of the selection of breaths defined by \( x \). This curve was obtained by repeating the simulation procedure 100 times and averaging the results. When breathing is at constant \( V_T \), the ratio of SD always provides a correct value for \( K \), regardless the value of \( x \), and thus a fortiori in the range \( 0.6 \leq x \leq 1 \).

**Approximation of Eq. 5 for simulated breathing at quasi-constant \( V_T \).** In the second simulation, some dispersion is added to \( V_T \) with respect to the ideal case of Fig. 2A. Figure 2B shows the distributions of the simulated data for this case, and Fig. 3B shows the corresponding Konno-Mead plot. It can be seen that \( \{V_T\} \) is no longer a Dirac distribution. Furthermore, the points are not perfectly aligned along a straight line of slope \(-1\). Again, the selection of breaths according to the criterion of Sackner (Eq. 5 with \( x = 0.6 \)) yields many breaths (55%) at quasi-constant \( V_T \) that are eliminated whereas breaths corresponding to a slope of \(-1\) remain. This is highlighted in Fig. 3B by the two dotted lines. According to the value of \( x \) and thus to the selected breaths, \( K \) will take different numeric values, which tend to 1 when \( x \) decreases (Fig. 4). In the present simulation, a selection of breaths with \( 0.6 \leq x \leq 1 \) yields a corresponding value \( K \) ranging from 0.938 and 0.864 instead of 0.7. Figure 5 shows \( K \) as a function of \( x \) for the four different cases of Table 2, i.e., with decreasing standard deviation of the rib cage and of the abdomen volume variations. Here the same averaging procedure as in Fig. 4 was performed. It can be seen that if the variability of the contributions of thoracic and abdominal compartments decreases, the accuracy of \( K \) decreases for constant \( x \).

**Approximation of Eq. 5 for simulated spontaneous breathing.** In the third simulation, we consider spontaneous breathing (Figs. 2C and 3C). In that case, the slopes of the calibrated and uncalibrated data in the Konno-Mead plot are positive. The values of \( K \) obtained

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**Fig. 2.** Thoracic and abdominal volumes (\( V_{rc} \) and \( V_{ab} \), respectively) generated by random sampling from a two-dimensional normal distribution (see Fig. 1) in 3 different cases. A: simulation of constant tidal volume (\( V_T \)). B: simulation of quasi-constant \( V_T \). C: simulation of spontaneous tidal breathing. Top: calibrated volumes (\( \Delta V \)). Bottom: uncalibrated volumes (\( \Delta uV \)). We assumed for all simulations a mean \( V_T \) of 0.75 liters and a mean abdominal relative contribution of 1/3. The sets of parameters used to construct the simulations are given in Table 1. \( Krc \) and \( Kab \) were chosen at 0.7 and 1, respectively. Every set contains 150 data points.


**DISCUSSION**

**Underlying theory and basic assumptions.** In this paper, we use the two-compartment model of Konno and Mead (Eq. 1), and we assume the validity of the observation of Sackner et al. (10) that $\Delta V_{rc}$ and $\Delta V_{ab}$ are normally distributed. This allowed us to focus on the mathematics underlying QDC. The idea of working at constant $V_T$ by analogy with the isovolume calibration is ingenious. We show in the APPENDIX that the formula for $K$ in the original paper (10) is correct although the mathematical demonstration can be criticized (3, 11, 13). In fact, we were intrigued by the negative sign of Eq. 4 and by the simplicity of the mathematical derivation of $K$ by taking the SD of each term of Eq. 2 and stating that the SD of a sum of statistical variables is equal to the sum of the SD of each variable. From a physiological point of view, it is clear that $K$ should be positive. In fact, when the thoracic and abdominal compartments are modeled by two cylinders and $\Delta uV_{ab}$ and $\Delta uV_{rc}$ are considered as variations of their cross-sectional areas, $K$ represents the ratio of the heights of each compartment, which are positive values. In the APPENDIX, it is shown that the QDC-calculated $K$ value can geometrically be interpreted as the approximation of the absolute slope of the regression line through the set of uncalibrated thoracic and abdominal volumes plotted in the Konno-Mead representation.

**Breath selection for constant $V_T$.** The critical point of the QDC procedure is the hypothesis of constant $V_T$ in situations in which this parameter is unknown. Sackner et al. (10) proposed to collect the breaths that fall in an interval around the mean $m(\Delta uV_{rc} + \Delta uV_{ab})$. We demonstrated that this selection is not needed for breathing at constant or quasi-constant $V_T$ (Figs. 3, A and B, and 4). In these cases, the selection procedure even biases the results by favoring in the Konno-Mead plot breaths that correspond to a slope of $-1$ instead of breaths that are at constant or quasi-constant $V_T$. When the respiratory pattern is spontaneous, $K$ takes a value that depends on the selection criterion and that tends to 1 with decreasing $x$. The latter observation is due to the selection procedure, which favors breaths falling on a line with a negative unitary slope. Hence, the breaths selected define a specific constant volume $V_T = \Delta uV_{rc} + \Delta uV_{ab}$ related to the situation of $K = 1$, which is not the actual value of $K$. Furthermore, it was

from QDC according to $x$ are shown in Fig. 4. Without any selection, $K$ equals $[0.7 \mu(\Delta V_{ab})/\mu(\Delta V_{rc})]$. For $0.6 \leq x \leq 1$, the values of $K$ range from 0.492 to 0.42. $K$ tends to 1 when $x$ decreases.

**Fig. 3.** Calibrated $\Delta V$ (×) and uncalibrated $\Delta uV$ (circles) thoracic and abdominal volumes of Fig. 2, plotted in a Konno-Mead representation. A: simulation of constant tidal volume $V_T$. B: simulation of quasi-constant $V_T$. C: simulation of spontaneous tidal breathing. The boundary lines with a slope of $-1$ delimit the selected breaths (○) corresponding to the interval $[\mu(\Delta uV_{rc} + \Delta uV_{ab}) \pm 0.6 SD(\Delta uV_{rc} + \Delta uV_{ab})]$. ●, Breaths outside the selection interval. Values of the different slopes are indicated by $\alpha$. See DISCUSSION for details.

**Table 1. Simulation parameters for Fig. 4**

<table>
<thead>
<tr>
<th>Respiratory Pattern</th>
<th>$\beta$</th>
<th>$SD(\Delta V_{rc})$</th>
<th>$SD(\Delta V_{ab})$</th>
<th>$SD(V_T)$</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>$-45$</td>
<td>40</td>
<td>40</td>
<td>0.1</td>
<td>$\times$</td>
</tr>
<tr>
<td>Quasi-constant</td>
<td>$-45$</td>
<td>40</td>
<td>40</td>
<td>20</td>
<td>$\triangle$</td>
</tr>
<tr>
<td>Spontaneous</td>
<td>26.6</td>
<td>40</td>
<td>20</td>
<td>60</td>
<td>$\square$</td>
</tr>
</tbody>
</table>

$\beta$, Angle of the major axis of the simulated thoracic and abdominal volumes with respect to the abscissa; $SD(\Delta V_{rc})$, $SD(\Delta V_{ab})$, and $SD(V_T)$, standard deviations of the thoracic, abdominal, and tidal simulated volumes, respectively. Symbols refer to the curves of Fig. 4.
shown that if the variability of the contributions of thoracic and abdominal compartments decreases, the accuracy of $K$ decreases for constant $x$. In fact, the selected points become more focused and more easily define a straight line with a slope corresponding to $K = 1$ instead of its real value. Finally, the limitation of the method used to keep breaths at constant VT is highlighted by the fact that multiplying the gain of one channel (rib cage or abdomen) does not induce the same variation of the QDC-calculated $K$. This observation is explained by the fact that the modification of gain induces a rotation of the points $(\Delta u_{\text{Vrc}}, \Delta u_{\text{Vab}})$ in the Konno-Mead plot while the selection interval of slope $(-1)$ remains identical. Hence, this operation modifies the breaths selected and thus the calculation of $K$.

Applications of QDC. In spite of these serious reservations, QDC was demonstrated to yield, in adults in supine posture, results equivalent to the other calibration procedures, namely isovolume and multilinear regression methods (10, 11). The best results were obtained when estimating VT during uninstructed tidal breathing. Other situations, such as natural preferential thoracic or abdominal breathing and voluntary

changes of VT, yielded less accurate VT estimations. These results can be explained by the observation that the variability of thoracoabdominal partitioning in normal adult subjects is very small during quiet breathing (2, 11). Hence the rib cage signal is proportional to the abdominal one. In that case, $\Delta u_{\text{Vrc}} = \eta \Delta u_{\text{Vab}}$ and Eq. 2 becomes

$$VT = M(1 + K\eta)\Delta u_{\text{Vab}}$$

It can be seen in Eq. 7, where $\eta$ is a proportionality constant, that a correct VT will be obtained with an appropriate choice of $M$, regardless of the value of $K$. This can explain why all the calibration techniques, including the one using standard ratios (2), provide a good evaluation of the VT for quiet breathing, whereas greater discrepancies in the evaluation of VT are observed when changes in breathing pattern occur. In the latter case, two degrees of freedom are used and, hence, the accuracy of $K$ becomes more critical.

Adams et al. (1) demonstrated the satisfactory use of the Respitrace, calibrated with QDC, for estimating VT in healthy full-term newborns. The validation was achieved shortly after QDC, by comparing the mean volumes of sets of 10 breaths obtained from calibrated signals with these measured by a pneumotachograph. The relative contributions of rib cage and abdominal compartments to VT were not examined, and isovolume-like situations (obstructive apneas or simulated obstructive apneas) were not analyzed. Hence, the accuracy of $K$ was not studied independently from the value of $M$. Furthermore, the values of $K$ and $M$ were not indicated in the paper of Adams et al., which prevents in-depth scrutiny of their results in the light of our analysis.

In contrast to the Adams et al. study (1), Brown et al. (3) showed QDC to be unreliable for estimation of the VT in anesthetized infants. To obtain sufficient accuracy when evaluating VT by QDC, a minimum of 20 breaths needed to be averaged in the analysis. In addition, the errors on the volume reconstructed by QDC during imposed airway obstructions were very large. Finally, the contribution of the rib cage to VT, calculated by means of the QDC-calibration factor, was unexpectedly high and inconsistent with published findings. The authors suggested that, in anesthetized infants, the contributions of rib cage and abdominal compartments during tidal breathing may not be sufficiently variable to allow for the accurate derivation of $K$. However, they could not verify this point due to the

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**Table 2. Simulation parameters for Fig. 5 at quasi-constant VT**

<table>
<thead>
<tr>
<th>$\beta$, $^\circ$</th>
<th>SD($\Delta u_{\text{Vrc}}$), ml</th>
<th>SD($\Delta u_{\text{Vab}}$), ml</th>
<th>SD(VT), ml</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-45$</td>
<td>40</td>
<td>40</td>
<td>5</td>
<td>△</td>
</tr>
<tr>
<td>$+45$</td>
<td>30</td>
<td>30</td>
<td>5</td>
<td>◆</td>
</tr>
<tr>
<td>$-45$</td>
<td>20</td>
<td>20</td>
<td>5</td>
<td>○</td>
</tr>
<tr>
<td>$+45$</td>
<td>10</td>
<td>10</td>
<td>5</td>
<td>+</td>
</tr>
</tbody>
</table>

Symbols refer to the curves of Fig. 5.
limitations of their experimental setup. Our analysis of the model at quasi-constant \( V_T \) confirms this hypothesis by showing that a decreasing variability in the contributions of thoracic and abdominal movements can actually make the QDC method fail (Fig. 5).

In conclusion, QDC will only provide a correct calibration factor when applied to an entire set of breaths with constant or quasi-constant \( V_T \). More generally, physiological conclusions based on QDC should be critically evaluated on a case-by-case basis.

**APPENDIX**

Demonstration of Eq. 4. Using \( \Delta V = V_T \) to consider tidal volumes, Eq. 2 can be rewritten as

\[
\Delta uVab = -K\Delta uVrc + \frac{V_T}{M} \quad (A1)
\]

Because \( V_T \) is constant in Sackner’s method (10), Eq. AI represents a straight line with a negative slope \(-K\) and intercept \( \frac{V_T}{M} \). In general, when pairs \((x_i, y_i)\) belong to a straight line \( y = ax + b \), the variance (SD\(^2\)) of the data sets \( X = \{x_i\} \) and \( Y = \{y_i\} \) are given by

\[
SD^2(X) = \frac{1}{n} \sum x_i^2 - \mu^2 \quad (A2)
\]

\[
SD^2(Y) = SD^2(aX + b) = \frac{1}{n} \sum (ax_i + b)^2 - (a\mu + b)^2 \quad (A3)
\]

\[
a^2 = \frac{1}{n} \sum x_i^2 - \mu^2
\]

where \( \mu \) is the mean of the set \( \{x_i\} \) and \( n \) is the number of data pairs \((x_i, y_i)\). Hence

\[
a = \frac{SD^2(Y)}{SD^2(X)} \quad (A4)
\]

By analogy, in the case of Eq. AI

\[
(-K)^2 = \frac{SD^2(\Delta uVab)}{SD^2(\Delta uVrc)} = K^2 \quad (A5)
\]

and thus, as \( K \) is positive, is

\[
K = \frac{SD(\Delta uVab)}{SD(\Delta uVrc)} \quad (A6)
\]

the solution of Eq. 2. \((-K)\) is interpreted geometrically as the slope of the line defined by the uncalibrated thoracic and abdominal volumes plotted in a Konno-Mead representation.

If the pairs \((x_i, y_i)\) do not belong to a straight line but are strongly correlated, a regression line can be calculated by a least-square fitting. If \( y \) is considered as the dependent variable, the slope is given by

\[
\alpha = \frac{Cov_{xy}}{SD^2(X)} = r \frac{SD(Y)}{SD(X)} \quad (A7)
\]

On the other hand, considering \( x \) as the dependent variable yields

\[
\beta = \frac{SD^2(Y)}{Cov_{xy}} = \frac{1}{r} \frac{SD(Y)}{SD(X)} \quad (A8)
\]

Here \( Cov_{xy} \) is the covariance of \((x_i, y_i)\) and \( r \) is the correlation coefficient. By analogy

\[
\alpha = r \frac{SD(\Delta uVab)}{SD(\Delta uVrc)} = rK \quad (A9)
\]

and

\[
\beta = \frac{1}{r} \frac{SD(\Delta uVab)}{SD(\Delta uVrc)} = \frac{1}{r} K \quad (A10)
\]

Equations A9 and A10 show that \((-K)\) calculated according to QDC is an approximation of the slope of the regression lines calculated on the uncalibrated thoracic and abdominal volumes plotted in the Konno-Mead representation. The approximation is all the better when the points are strongly negatively correlated, i.e., when \( r \) tends to \(-1\).

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