Influence of upper airway shunt on total respiratory impedance in infants

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Desager, K. N., M. Cauberghs, J. Naudts, and K. P. Van de Woestijne. Influence of upper airway shunt on total respiratory impedance in infants. J. Appl. Physiol. 87(3): 902–909, 1999.—When input impedance is determined by means of the forced oscillation technique, part of the oscillatory flow measured at the mouth is lost in the motion of the upper airway wall acting as a shunt. This is avoided by applying the oscillations around the subject’s head (head generator) rather than at the mouth (conventional technique). In seven wheezing infants, we compared both techniques to estimate the importance of the upper airway wall shunt impedance (Zuaw) for the interpretation of the conventional technique results. Computation of Zuaw required, in addition, estimation of nasal impedance values, which were drawn from previous measurements (K. N. Desager, M. Willemen, H. P. Van Bever, W. De Backer, and P. A. Vermeire. Pediatr. Pulmonol. 11: 1–7, 1991). Upper airway resistance and reactance at 12 Hz ranged from 40 to 120 and from 0 to −150 hPa·l⁻¹·s⁻¹, respectively. Varying nasal impedance within the range observed in infants did not result in major changes in the estimates of Zuaw or lung impedance (ZL), the impedance of the respiratory system in parallel with Zuaw. The conventional technique underestimated ZL, depending on the value of Zuaw. The head generator technique slightly overestimated ZL, probably because the pressure gradient across the upper airway was not completely suppressed. Because of the need to enclose the head in a box (which is not required with the conventional technique), the head generator technique is difficult to perform in infants.

When the impedance of the respiratory system (Zrs) is determined by applying forced oscillations at the mouth (8), the upper airway acts not only as an impedance in series with the lungs but also as a shunt for the oscillatory flow. Motion of the upper airway wall results in loss of flow (V˙), leading to an underestimation of the impedance of the downstream respiratory system. In 1956, DuBois et al. (8) already recommended support of the cheeks with the palms of the hands to reduce the motion of the cheeks. In an attempt to correct for upper airway wall shunt impedance (Zuaw) in adults, Michaelson et al. (14) performed measurements during a Valsalva maneuver and subtracted the obtained impedance value from the Zrs. This correction overestimates the influence of the upper airway on Zrs. Indeed, during the Valsalva maneuver the upper airway behaves as a single shunt impedance, with the series element of the upper airway being negligible (1, 16). Besides, performance of a Valsalva maneuver requires good cooperation, which cannot be obtained in infants. As an alternative, Peslin et al. (15) tried to reduce the influence of the upper airway wall shunt on respiratory impedance measurements by using a head generator in which pressures are varied around the head rather than directly at the mouth (conventional technique), thereby minimizing transmural pressure across the upper airway wall. Comparison of respiratory impedance measured with the head generator or by the conventional technique in adults yielded values of upper airway impedance that were similar to those measured directly with a head plethysmograph (16).

In 1989, Marchal et al. (12) compared the use of the head generator with the conventional technique in 24 infants. Because infants are nearly obligatory nose breathers, the assumption that upper airway wall motion is eliminated because oscillations are applied around the head is not entirely valid. Indeed, during nose breathing the nose induces a pressure difference across the upper airway wall, resulting in upper airway wall motion. This problem was recognized by the authors, and an adaptation of their initial model was introduced in the discussion, enabling them to simulate the influence of the upper airway in nose-breathing infants. No attempt thus far has been made to directly estimate Zuaw in infants.

The aim of the present study was to estimate Zuaw in infants by means of a model combining impedance measurements obtained with the conventional forced oscillation technique and with the head generator and pressure measurements obtained with a head plethysmograph. Estimation of Zuaw allowed us to evaluate the validity of results of conventional and head generator techniques.

Materials and Methods

Nine infants, hospitalized with bronchiolitis, were recruited for the study. Their ages varied between 4 and 17 mo. The study was approved by the local hospital ethics committee, and oral informed consent was obtained from the parents.

Conventional technique combined with head plethysmograph. A schematic representation of the experimental setup is presented in Fig. 1. A volume-constant head plethysmograph was made of a 35 x 20 x 24-cm wooden box. Two walls consisted of Plexiglas to ensure visualization of the baby. One sidewall consisted of two parts, which delimited an orifice 10 cm in diameter. After sedation with oral choral hydrate in a dose of 80 mg/kg, the infant was nursed supine, and the two sidewall parts were assembled around the lower part of the neck, thus closing the box. A reasonably good seal with

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minimal compression of the airways was obtained by covering the space between the orifice and the neck with a rim of silicone putty. A 90-W loudspeaker, mounted in the wall of a chamber, generated pseudorandom noise containing all harmonics of 4 up to 52 Hz. The oscillations were conducted to the infant through a 60-cm-long and 2-cm-bore flexible tube, and a well-fitting rigid face mask with a silicone border was adapted to each infant individually (5). Differential pressure across a Fleisch no. 1 pneumotachograph, yielding $V\dot{}$, airway opening pressure ($P_{ao}$) measured as close as possible to the face mask, and box pressure ($P_{box}$) were measured with identical differential pressure transducers ($\pm 2$ hPa, Validyne MP45–1). $P_{box}$, $P_{ao}$, and $V\dot{}$ were digitized with a sampling rate of 128 Hz and fed into a computer. The ratio of $P_{ao}$ to $V\dot{}$ ($P_{ao}/V\dot{}$) was calibrated by using two reference loads and according to the two-point calibration procedure (6).

The impedance value, calculated from $P_{ao}/V\dot{}$, corresponds to that measured with the conventional forced oscillation technique.
technique and is called \( Z_1 \) (resistance, \( R_1 \); reactance, \( X_1 \)). The ratio of \( \text{Pbox to Pao} \) (\( \text{Pbox/Pao} \)) will be referred to as a “transfer function” (TF) with a real and imaginary part.

To get information on the impedance of the box (\( Z_{\text{box}} \)), the face mask was removed from the infant's face without opening the box or moving the infant, and \( \text{PaoV} \) was determined. The impedance values obtained were called \( Z_3 \) (resistance, \( R_3 \); reactance, \( X_3 \)).

Head generator technique. The experimental setup was easily changed into a head generator (15) by disconnecting the tube from the pneumotachograph, thus allowing the infant to breathe into the box (Fig. 2). Oscillations produced by the loudspeaker were now applied around the head, and the impedance values obtained were called \( Z_2 \) (resistance, \( R_2 \); reactance, \( X_2 \)).

During this procedure of changing from the conventional technique to the head generator, the infant was not moved, the head was not tilted, and the face mask was left in place. Measurements of \( Z_1 \), \( Z_2 \), and \( Z_3 \) were performed in a random order, and at least four reproducible values with a coherence function \( \geq 0.95 \) were obtained for each parameter in each infant. Because of a lower signal-to-noise ratio, measurements \( < 12 \) Hz were generally not satisfactory (coherence function \( < 0.95 \)).

Estimation of \( Z_{\text{zuaw}} \). The experimental setup allows for the determination of four variables: \( Z_1 \), \( Z_2 \), \( Z_3 \), and TF. In the conventional technique measuring \( Z_3 \), \( Z_{\text{zuaw}} \) is assumed to be in parallel with the impedance of the lungs, airways, and chest wall (which will be referred to as \( Z_1 \)), whereas, when oscillations are applied around the head, yielding \( Z_2 \), \( Z_{\text{zuaw}} \) is in parallel with nasal (\( Z_n \)) and pneumotachograph impedance (\( Z_p \)). Figure 3 depicts electrical analogs for the three measurement conditions, which can be described by the following equations

\[
Z_1 = Z_n + Z_1 \cdot (Z_{\text{zuaw}} + \text{Zbox})/(Z_1 + Z_{\text{zuaw}} + \text{Zbox})
\]

(1)

\[
Z_2 = (Z_1 - Z_{\text{zuaw}}) + Z_1 \cdot (Z_p + Z_{\text{zuaw}})/Z_{\text{zuaw}}
\]

(2)

\[
\text{TF} = \frac{Z_2 - Z_1}{Z_{\text{zuaw}}}
\]

(3)

Combining Eqs. 1-3, we can solve for

\[
Z_{\text{zuaw}} = \alpha \cdot Z_1 - \text{Zbox}
\]

(4)

\[
Z_n = Z_1 - \frac{[\text{Zbox} \cdot (\alpha + 1) \cdot (Z_2 - Z_{\text{zuaw}})]}{[Z_2 \cdot (\alpha^2 + 1) - Z_1 \cdot (\alpha^2 + 2\alpha + 1) - Z_p \cdot (\alpha + 1) + \text{Zbox}]}
\]

(5)

where \( \alpha = (\text{Zbox} - \text{TF} \cdot Z_1) / (\text{TF} \cdot Z_1) \).

In addition to the values of \( Z_1 \), \( Z_2 \), and TF measured directly, the solution of Eqs. 4-6 requires an estimate of \( \text{Zbox} \), which was derived from the setup yielding \( Z_3 \). Initially, as a first approximation, we assumed that, in the \( Z_3 \) setup, the arm of the model that consisted of the combination of \( Z_n \), \( Z_{\text{zuaw}} \), and \( Z_1 \) had a very high impedance value with respect to the other arm that consisted of \( \text{Zbox} \) (Fig. 3). In that case, \( Z_3 \) was a good estimate of \( \text{Zbox} \). \( Z_1 \), \( Z_{\text{zuaw}} \), and \( Z_n \) were then calculated from Eqs. 4-6 by using the measured values of \( Z_1 \), \( Z_2 \), \( Z_3 \), and TF.

As a check for the validity of the mathematical model, loads made of sintered glass fitted in glass tubes (5), with known impedance values, were connected as a model of lungs, upper airway, and nose.

Because the results obtained by using the approximate expression of \( \text{Zbox} \) did not yield satisfactory results (see RESULTS), we decided to work out the exact relation between \( Z_3 \) and \( \text{Zbox} \). It is

\[
Z_3 = \frac{\text{Zbox} \cdot (Z_1 \cdot Z_n + Z_{\text{zuaw}} + Z_1 \cdot Z_{\text{zuaw}})}{(Z_1 \cdot Z_n + Z_{\text{zuaw}} + Z_1 \cdot Z_{\text{zuaw}} + \text{Zbox} \cdot Z_n)}
\]

(7)

It turned out that Eqs. 1-3 and 7 are not independent. Indeed, it can be shown that

\[
aZ_1^2 \text{TF}^2 + bZ_1^2 \text{TF} + c = 0
\]

(8)

where

\[
a = Z_3 - Z_2
\]

\[
b = Z_p (Z_3 - Z_2) + Z_2 (Z_3 - Z_2) - 2Z_1 Z_3 + Z_1 Z_2
\]

\[
c = Z_1 Z_3 (Z_1 - Z_2)
\]

It is impossible to determine the four unknowns, \( Z_{\text{zuaw}} \), \( Z_L \), \( Z_n \), and \( \text{Zbox} \), from the four measurements from which only three are independent. Eqs. 4-6 depend strongly on an accurate measurement of the parameter \( \alpha \). It involves TF, which is very small, because the \( P_2 \) pressure point is low with respect to the \( P_1 \) pressure point (see Fig. 3 and Table 1). Therefore, we decided not to use Eq. 3 and to introduce into the model, as an independent variable, values of \( Z_n \), which had been obtained previously in 30 infants by the forced oscillation technique by combining consecutive measurements through both nostrils and each nostril separately (7).

The mean values at 24 Hz of nasal resistance (\( R_n \)) and reactance (\( X_n \)) in 23 asthmatic infants without nasal obstruction were \( 2.5 \pm 2 \) and \( -0.9 \pm 3.7 \) hPa \( \cdot \) l/s, respectively. These values \( +1 \) SD and \(-1\) SD were used as representative values. This results in the following values: \( R_n \), 0.5, 2.5, and 4.5 hPa \( \cdot \) l/s; \( X_n \), -4.6, -0.9, and 2.8 hPa \( \cdot \) l/s. \( Z_{\text{zuaw}} \), \( Z_L \), and \( Z_{\text{zuaw}} \) were then calculated from \( Z_1 \), \( Z_2 \), \( Z_3 \), \( Z_n \), and \( Z_p \), as shown in the APPENDIX.

RESULTS

Measurements were extremely difficult to perform because the sedated baby had to be nursed supine with his or her head in a tightly closed box with minimal leakage. Reproducible impedance values for the three experimental conditions were obtained in seven infants (Table 1). The data of two infants were excluded: in one patient leakage of the mask resulted in abnormally low resistance values, and in another patient \( Z_3 \) measurements showed major variations between frequencies. Because these variations were reproducible and the coherence function was \( \geq 0.95 \), this phenomenon was probably due to cross talk between frequencies because of marked nonlinearities of \( Z_r \)s (4). Mean SDs were lower with the conventional technique than with the head generator, especially at higher frequencies.

When \( Z_1 \), \( Z_2 \), \( Z_3 \), and TF were substituted into Eqs. 4-6, values of \( Z_n \) were obtained nearly equal to \( Z_1 \), whereas \( Z_L \) and \( Z_{\text{zuaw}} \) were close to zero. Similar results were obtained with the physical analog. These
results are not physiologically meaningful, and thus the assumption that $Z_3 = Z_{\text{box}}$ is invalid.

On the other hand, the results were different from zero and made sense when the exact relation for $Z_{\text{box}}$ was used (Figs. 4 and 5; see DISCUSSION).

Varying the values of $R_n$ or $X_n$ did not result in major differences in $Z_{\text{aw}}$ or $Z_L$. In most infants, $Z_n$ did not show a frequency dependence, with the maximum range of $R_n$ and $X_n$ being, respectively, 4.6 and $-4.0 \text{ hPa} \cdot \text{l}^{-1} \cdot \text{s}$ at 12 Hz and 1.2 and 1.6 $\text{hPa} \cdot \text{l}^{-1} \cdot \text{s}$ at 52 Hz. Introduction of this frequency dependence of $Z_n$ in the model did not result in marked differences in the estimates of $Z_{\text{aw}}$ or $Z_L$.

When estimated $Z_L$ was compared with $Z_1$ and $Z_2$, two different patterns were observed: lung resistance ($R_L$) independent of frequency ($n = 4$) and $R_L$ increasing with increasing frequency (positive frequency dependence) ($n = 3$). The values of lung reactance ($X_L$) initially increased and then decreased at the higher

![Fig. 3. Electrical circuits for the three measurement conditions: A, conventional technique; B, conventional technique with baby disconnected, showing estimation of box impedance ($Z_{\text{box}}$); C, head generator technique. $Z_p$, impedance of pneumotachograph; $Z_n$, nasal impedance; $Z_{\text{aw}}$, upper airway wall impedance; $Z_L$, impedance of chest, airways, and chest wall; $P_1$, $P_2$, $P_3$, and $P_4$, pressures at different points in the circuits; $V_1$, $V_2$, and $V_3$, flows at different points in the circuits. See text for details.](image)

![Fig. 4. Resistance (A) and reactance (B) of upper airway in 7 infants (each represented by different symbol).](image)

### Table 1. Measurements obtained with conventional technique combined with head plethysmograph and head generator technique

<table>
<thead>
<tr>
<th>Patient No.</th>
<th>$R_1$</th>
<th>$R_3$</th>
<th>$R_2$</th>
<th>$R_3$</th>
<th>$TF_{\text{real}}$</th>
<th>$R_1$</th>
<th>$R_3$</th>
<th>$R_2$</th>
<th>$R_3$</th>
<th>$TF_{\text{real}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>39.7</td>
<td>57.2</td>
<td>0.28</td>
<td>0.001</td>
<td>29.8</td>
<td>55.2</td>
<td>0.30</td>
<td>-0.005</td>
<td>-0.005</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>23.7</td>
<td>29.4</td>
<td>0.20</td>
<td>0.0002</td>
<td>26.5</td>
<td>78.4</td>
<td>0.22</td>
<td>-0.008</td>
<td>-0.008</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>27.1</td>
<td>47.8</td>
<td>0.20</td>
<td>0.001</td>
<td>16.4</td>
<td>83.5</td>
<td>0.30</td>
<td>-0.007</td>
<td>-0.007</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>30.6</td>
<td>44.9</td>
<td>0.12</td>
<td>0.003</td>
<td>23.0</td>
<td>79.5</td>
<td>0.45</td>
<td>-0.006</td>
<td>-0.006</td>
<td></td>
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<tr>
<td>5</td>
<td>22.7</td>
<td>53.7</td>
<td>0.35</td>
<td>0.004</td>
<td>18.1</td>
<td>55.7</td>
<td>0.20</td>
<td>-0.005</td>
<td>-0.005</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>31.8</td>
<td>47.3</td>
<td>0.24</td>
<td>0.001</td>
<td>22.5</td>
<td>50.7</td>
<td>0.31</td>
<td>-0.003</td>
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</tr>
<tr>
<td>7</td>
<td>40.4</td>
<td>50.6</td>
<td>0.63</td>
<td>0.004</td>
<td>32.4</td>
<td>77.2</td>
<td>0.23</td>
<td>-0.005</td>
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Measurements of the real and imaginary parts of the impedance [resistance ($R$) and reactance ($X$); in $\text{hPa} \cdot \text{l}^{-1} \cdot \text{s}$] and transfer function ($TF$) obtained with the conventional technique (denoted by subscript 1) combined with the head plethysmograph (denoted by subscript 3; $R_1$, $R_3$, $TF_{\text{real}}$) and $X_1$, $X_3$, $TF_{\text{imag}}$, respectively) and head generator technique (denoted by subscript 2; $R_2$ and $X_2$) at 12 and 52 Hz for 7 infants. Mean SD is for 7 infants.
frequencies. An example of both patterns is shown for \( Z_1, Z_2, \) and \( Z_L \) (Figs. 8 and 9). There was a close parallelism between \( Z_L \) and \( Z_2 \), with the values of \( R_L \) and \( X_L \) being slightly less than the corresponding values of \( R_2 \) and \( X_2 \). The difference between \( Z_L \) and \( Z_1 \) was larger for both \( R_L \) and \( X_L \). As shown in Fig. 10, this difference depends on the ratio of upper airway wall resistance (\( Ru_{aw} \)) to \( R_L \) (\( Ru_{aw}/R_L \)).

**DISCUSSION**

By combining measurements obtained with the conventional forced oscillation technique, the head generator, and previously determined \( Z_n \) values, it was possible to estimate \( Zu_{aw} \) and \( Z_L \) in infants.

Previously, consecutive measurements of the conventional (without head plethysmograph) and head generator techniques have been performed by Marchal et al. (12) in 24 infants, aged 2–49 mo. Data were analyzed between 6 and 20 Hz only because the reproducibility of the measurements was poor >20 Hz. In our measurements, a satisfactory reproducibility was obtained up to 52 Hz, although SDs were higher in the higher frequency range with the head generator. Resistance values obtained with the head generator were higher than with the conventional technique, whereas reactance values were on the average less negative. Our data are in agreement with the findings of Marchal et al. and with the previous observations of our laboratory (5).

Several investigators have reported on the mechanical characteristics of the upper airway in adults (1–3, 9, 11, 14). Ruaw varied between 4 and 10 hPa·l\(^{1-1}\)·s, whereas upper airway wall reactance (\( Xu_{aw} \)) ranged between \(-2.1\) and \(-5.3\) hPa·l\(^{1-1}\)·s for frequencies between 4 and 28 Hz, depending on whether or not the cheeks were supported. Cauberghs and Van de Woestijne (1, 2) found Ruaw values of \(~20\) hPa·l\(^{1-1}\)·s at 4 Hz, decreasing to 10 hPa·l\(^{1-1}\)·s at 20 Hz, with a Xuaw of \(-51\) and \(-11\) hPa·l\(^{1-1}\)·s at 4 and 20 Hz, respectively, in adults. These data were confirmed by the measurements with the head plethysmograph (16).

In children, few data are available. In 15 children, aged 5–15 yr, Ruaw with support of the cheeks was found to be 46 hPa·l\(^{1-1}\)·s at 4 Hz, decreasing to 21 hPa·l\(^{1-1}\)·s at 20 Hz, whereas the reactance was \(-65\) and \(-20\) hPa·l\(^{1-1}\)·s at 4 and 20 Hz, respectively (1). There are no data available on the upper airway impedance in infants. Our data show still higher resistance and more negative reactance values in some infants than in children. This is to be expected from the comparison of data between adults and children.

Because infants breathe through the nose, it seemed that the head generator did not correctly take the upper airway shunt into account. Marchal et al. (12) recognized this problem and estimated the error that results
from either techniques. Two assumptions were made: first, the nose was assumed to represent 40 and 20% of total respiratory system resistance and inertance, respectively; and, second, Ruaw and Xuaw were chosen to be 80 and 77.1 hPa·l⁻¹·s⁻¹, respectively. With the use of these hypothetical values for Zn and Zuaw, the Zrs without any shunt effects was calculated. Marchal et al. found that, with the head generator technique, resistance was overestimated by 6%, increasing to 8% in airway obstruction, whereas with the conventional technique the underestimation was 13% and was 23% in airway obstruction. In the present study, we used previously determined Zn values to estimate the upper airway shunt. Alternatively, one nostril was occluded, and measurements were performed through the other. By combining these two measurements with impedance values obtained with the unoccluded nose, Zn was calculated (7). These measurements were performed in a group of children other than those in the present study. Indeed, it was practically not possible to perform these occlusions in the setup of the present study: the infant was lying with his or her head fixed in a box, which remained closed during the measurement of Z₁, Z₂, Z₃, and TF. However, because the study of Zn showed that the range of values was small as long as the nose was not obstructed by secretions, we felt that these values could be used to solve the set of equations in the present study. This resulted in estimations of Z₁ closer to Z₂ (head generator) than to Z₁ (conventional technique), with the underestimation of Z₁ being larger with smaller Ruaw/RL. If Zuaw is high, less flow goes to the shunt pathway and thus Z₁ reflects more closely Z₁. The overestimation of RL with the head generator varied between 6 and 18% (mean 10%) at 20 Hz, whereas with the conventional technique the underestimation ranged from −13 to −57% (mean: −35%). Errors in reactances, expressed in absolute values, were greater for the conventional than for the head generator technique.

Comparative studies of the head generator and the conventional technique have been performed in adults (2, 10) and in school-age children (13). In normal adult subjects, measurements obtained with both techniques are very similar. In patients with moderate airway obstruction, a negative frequency dependence is observed with the conventional technique, whereas resistance is higher without frequency dependence with the head generator. In patients with severe obstruction, both techniques show resistance values with negative frequency dependence, although resistance values are higher with the head generator. Accordingly, a negative frequency dependence of resistance may be an artifact due to the upper airway shunt only in patients with moderate airway obstruction. Moreover, this artifact is not necessarily an inconvenience for diagnostic purposes, because it allows a clear-cut separation between

![Fig. 7. Influence of different nasal resistance and reactance values (in hPa·l⁻¹·s⁻¹) on mean values of respiratory resistance (A) and reactance (B). See Fig. 6 legend for explanation of symbols.](http://jap.physiology.org/)
In conclusion, Zuaw was estimated by using a combination of measurements obtained with the conventional forced oscillation and head generator techniques and previously determined Zn values. Upper airway resistance and reactance at 12 Hz were between 40 and 120 and 0 and –150 hPa·l⁻¹·s⁻¹, respectively. Head generator impedance values are closer to the estimated ZL values than are those of the conventional technique. Still, the head generator technique slightly overestimates the Zrs, because it does not completely suppress the pressure gradient across the upper airway. Because the head of the infant has to be enclosed in a box, the technique is unpractical and nearly impossible to perform routinely.

Appendix

From Eqs. 1, 2, and 7, it follows that Zuaw satisfies the following quadratic equation

\[ y_0(Zuaw)^2 + y_1Zuaw + y_0 = 0 \]  \hspace{1cm} (A1)

with

\[ y_0 = (Z_2 - Z_3)(Z_1 - Z_2) \]
\[ y_0 = (Z_1 - Z_n)(Z_2 - Z_3)/Z_1 - Z_p - (Z_2 - Z_1)(Z_2 - Z_p)/Z_1 - Z_n \]

Given the largest of the two solutions of Zuaw in Eq. A1, one calculates Zbox from

\[ Zbox = Zuaw(Z_2 - Z_1)Zuaw - (Z_1 - Z_n)(Z_2 + Z_p) \]

and \( Z_1 \) from

\[ Z_1 = Zuaw(Z_2 - Z_n)/(Z_2 - Z_p + Zuaw) \]  \hspace{1cm} (A3)

Finally, note that Eq. 8 can be written as

\[ Z_1TF = Zbox(Z_1 - Z_n)/(Zbox + Zuaw) \]  \hspace{1cm} (A4)

Eq. A4 justifies the choice of the largest solution of Zuaw. Indeed, the observed small values of TF require taking the largest solution of Eq. A1 for Zuaw.
REFERENCES