Modeling of adaptations to physical training by using a recursive least squares algorithm

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The present study investigates the usefulness of a model with parameters free to vary over time when using a recursive least square algorithm (16). The purpose is to assess whether the expected reduction of the residuals after fitting this time-varying model would provide further information on the adaptations to physical training. The models with parameters constant and varying over time were compared by using data collected from stressful training on a cycle ergometer by two volunteers.

BASIC FRAMEWORK

The model described herein is based on a systems model initially proposed by Banister et al. (4). The subject is represented by a system, the input of which is the daily total exercise level and time and the output is the performance. The working of the system is described by a transfer function, which is the sum of two first-order transfer functions. The impulse response (gt) of such a function is

\[ g(t) = k_1 e^{-\frac{t}{\tau_1}} - k_2 e^{-\frac{t}{\tau_2}} \]  

where \( k_1 \) and \( k_2 \) are gain terms (\( k_1 < k_2 \)), and \( \tau_1 \) and \( \tau_2 \) are time constants (\( \tau_1 > \tau_2 \)). The model performance is obtained by convolving the training doses, quantified from exercise level and time, to the impulse response (gt). The model parameters are determined by fitting the model performances to actual performances by the least square method. \( k_1 \) and \( k_2 \) were in arbitrary units, depending on the units used to measure the training load and performance in our studies (7–10, 21), whereas in studies of Banister’s group (3–6, 20) \( k_1 \) and \( k_2 \) were dimensionless. Only the ratio \( k_2/k_1 \) can be thus compared between studies. \( t_n \) is defined as the time needed after an impulse training stimulus for the effects of fatigue to be dissipated sufficiently to allow the effects of training to return performance to the pretraining level (13, 20). Thereafter, the performance will exceed its pretraining level. \( t_n \) is estimated by

\[ t_n = \frac{\tau_1\tau_2}{\tau_1 - \tau_2} \left( \ln \frac{k_1}{k_2} \right) \]

The \( t_n \) was 23 days for an elite hammer thrower who trained once or twice a day and had trained for 7 yr (7). This was greater than the 8 and 11 days reported for two subjects following an intensive program of running 40–50 min at least once each day for 28 days (20). The lowest values for \( t_n \) were 1–3 days for eight subjects performing a moderate endurance
training of four 1-h sessions at 60–90% of maximal oxygen uptake (VO2max) on a cycle ergometer per week for 14 wk (8). These differences were illustrated by computing the time impulse response g(t) for three subjects: subject S5 showing the higher value for k3 in Ref. 8, subject RHM in Ref. 20, and the athlete studied in Ref. 7. The time impulse response for these three subjects for an exercise leading to a decrease in performance of 1 arbitrary unit one day after the training completion is shown in Fig. 1. The greater the training intensity, the greater the time needed to recover from a single training session. Difference in the time course of the performance recovery arose, on the one hand, from a difference in \( \tau_3 < 3 \text{ days} \) in subjects undergoing moderate training (8), and, on the other hand, from a difference in the \( k_2/k_1 \) ratio: 4 in the athlete (7) vs. 1.8 and 2 in subjects undergoing intensive training (20), since \( \tau_2 \) was similar in these two studies, 13 days (7) and 11 days (20).

The observed differences in the published model parameters could arise in part from differences in training intensity. A highly trained subject should not need a longer time for recovery than an untrained one. However, an elite athlete would need to train greatly more than an untrained one for an equivalent gain in performance. This greater training demand is also likely to lead to increased fatigue. The repetition of exercises could alter the responses to a given training dose. If a session occurs before complete recovery from the preceding one, the negative effect could be amplified, and the regeneration time needed would be longer than if there was a longer gap between training sessions. The model parameters estimated for moderate training in Ref. 8 are unlikely to be representative of the responses of the same subjects to more strenuous training, whereas the model parameters estimated for the athlete in Ref. 7 could not reflect his response to a single training session.

The differences in observed \( t_n \) would be in keeping with data related to overtraining. Short-term overtraining for a few days to 2 wk is differentiated from long-term overtraining lasting weeks and months (14, 17, 18). The mechanisms underlying overtraining are not clear. The fatigue defined as a failure to maintain an expected power output (15) could be due to a combination of factors, including electrophysiological, metabolic, and ionic processes that occur within < 1 day (12, 15, 19). Short-term overtraining has been generally associated with incomplete restoration of cellular homeostasis between two training sessions due to the accumulation of waste products and depletion of muscle substrate (14, 17). Both the accumulation of waste products and glycogen depletion could lead to structural muscle damage (1, 2). Long-term overtraining has been associated with impairment of the neuroendocrine, autonomic, and immunological systems and mood state from which recovery could require weeks or months (14, 17, 18). The transient decreases in performance as a result of intensive training could thus arise from a mosaic of physiological processes that would have different dynamics. As the model described above is rather a model of data (11) or an empirical model (22), it summarizes the behavior of the performances during training. The model parameters would thus reflect the essential physiological mechanisms responsible for the observed performance response. The \( t_n \) values observed in modeling studies would thus be linked to the importance of the physiological processes involved in each kind of experiment. When the statistical analysis showed only one component, this may correspond to an extreme case. Only one positive component might correspond to subjects in whom fatigue was negligible, as opposed to performance gain during moderate training in low-fitness subjects (8).

Therefore, the linear time-invariant functions used in the model could be unsuitable for describing responses to training with varied regimens. The model parameters were assumed to be constant throughout the experimental period in the above-cited studies. However, day-to-day variations in model parameters, which would lead to a better fit of the performances, might describe more precisely adaptations to training and long-term fatigue. The present study investigates the relevance of a model with parameters (gain terms and time constants) free to vary over time, using a recursive least square method.

**EXPERIMENTAL DATA**

Protocol. Two subjects, A and B, were volunteers for the present study. The two subjects, aged 36 and 29 yr, were recreational cyclists, with VO2max of 44.4 and 45.4 ml·min\(^{-1}\)·kg\(^{-1}\), respectively, before the study. They trained regularly on a cycle ergometer (Monark) during the 4 mo preceding the experiment, with 2–4 sessions/wk, with intermittent exercises identical to those used during the experiment. This pretraining period was carried out to get the subjects used to the training conditions and to control their fitness status at the beginning of the experiment. The 14-wk experiment included two 4-wk periods of intensive training (weeks 1–4 and 7–10), separated by a 2-wk period of reduced training (weeks 5 and 6). The last 4 wk of the study were also a period of reduced training (weeks 11–14). The subjects performed every exercise on a cycle ergometer (Monark). The power generated during exercise was displayed by a system integrating the speed of the free wheel and the braking force.

During the two periods of intensive training (weeks 1–4 and 7–10), the subjects trained 5 consecutive days followed by 2 days of rest. However, subject A did not comply with the protocol during week 3. During the periods of reduced training (weeks 5, 6, and 11–14), the training frequency was reduced to 3–4 sessions/wk, and the training loads were halved. The training sessions included four kinds of intermittent exercises: S2–2 with 2 min of work interspersed with 2 min of active recovery, S3–3 with 3 min of work and 3 min of...
recovery, S5–3 with 5 min of work and 3 min of recovery, and S10–5 with 10 min of work and 5 min of recovery. During the periods of intensive training, these sequences were repeated 10 times for S2–2 and S3–3, 8 times for S5–3, and 4 times for S10–5. Furthermore, the sessions S2–2, S3–3, and S5–3 were composed of two bouts of equal duration elapsed by 15 min of rest. During the periods of reduced training, the number of repetitions was halved and included in a single exercise bout. The power maintained during exercise and active recovery was adjusted by the subjects. Exercise intensities were prescribed at 100% of maximal aerobic power (MAP) for S2, 95% for S3, 90% for S5, and 85% for S10. However, to keep a good compliance of the subjects, they were free to adapt day to day the exercise intensity according to their fatigue and/or fitness status.

The subjects performed every other week, an incremental test until exhaustion to measure the VO2max and the external MAP. The subjects warmed up and then exercised for 6 min at a work rate corresponding to ~60% of VO2max, the work rate was then increased every 2 min by 30 or 40 W, depending on the subject, until exhaustion. The subjects breathed through a two-way non-rebreathing valve (Hans Rudolph). The expired gases were collected in a mixing chamber connected to gas analyzers (Ametek S-3AI for O2 tension and Normocap Datex for CO2 tension). The gases were collected in a Tissot spirometer for measuring minute ventilation. MAP was calculated as the product of VO2max and the net efficiency, estimated from the 6-min exercise at constant load. Net efficiency was computed as the ratio between VO2max exceeding basal metabolic rate and external power output.

A constant-load exercise was performed until exhaustion before most of the training sessions, three to five times each week of intensive training, and one to three times each week of reduced training. The subjects performed these trials separately and were verbally encouraged. The maximal duration sustained by the subjects was recorded and used as a criterion of performance (Tmax). The load used in these trials was fixed before the experiment and maintained during it. It was chosen to provide a Tmax close to 5 min at the beginning of the study. After a first evaluation during the week before the experiment, the load was fixed to 270 and 360 W in subjects A and B, respectively. Indeed, Tmax was 5 min for subject A and 4 min 40 s for subject B on the first day of the experiment.

Quantification of training. The total duration of exercise and the mean power sustained were registered every day. However, the training sessions differed in the duration of each exercise and the recovery between them. The training sessions were compared by considering that each of them corresponded to the same training dose when exercises were performed at the prescribed intensity. The work executed during the recovery was not considered in the computation of the training doses. A training session would correspond to 100 or 50 units, respectively, for periods of intensive and reduced training, when it was performed at the reference intensity: 100% of MAP for S2–2 session, 95% for S3–3 session, 90% for S5–3 session, and 85% for S10–5 session. The training dose was then corrected with respect to the true exercise intensity. For example, a training session S3–3 at a mean intensity of 90% of MAP during intensive training would correspond to a training dose of 100 multiplied by the ratio between 90 and 95 = 94.7 units. The training dose corresponding to the trial to determine MAP was fixed at 20 units. Furthermore, a trial performed at 100% of MAP giving a Tmax of 5 min was also considered to be equal to 20 units. The training doses corresponding to the Tmax exercise were referred to these values according to their actual duration and intensity. For example, a Tmax exercise lasting 7 min and performed at 95% of MAP would give a training dose equal to 20 units multiplied by the ratio between actual and reference duration (7/5) and intensities (95/100), giving a dose of 26.6 units.

Variations in performance. Both subjects were exposed to a level of exercise stress much greater than those to which they were accustomed. The adaptive responses to the training stimulus produced large gains in Tmax in both subjects. However, the repetition of the training loads also induced transient decreases in performance.

The plot of the best performance reached during each week of the experiment shows the improvement in subjects’ fitness (Fig. 2). However, the rate of increase in performance was not steady during the experiment. After an initial increase in week 1, the performance improved slowly until week 4 only in subject A. Then, the two subjects increased their performance with the 2 wk of reduced training (weeks 5 and 6). The gains in performance were more substantial during the first 3 wk of the second period of intensive training (weeks 7–9). This progression was interrupted in week 10. The imbalance between exercise and recovery resulted in a severe, prolonged fatigue at the end of the second period of intensive training. The subjects complained of muscle soreness and had great difficulty complying with the training program from weeks 9 to 11. The performances of both subjects were noticeably better than on week 9 only on week 13, during the period of reduced training.

On the other hand, the training regimen resulted in a short-term transient decrease in performance. A decrease in performance was observed with the 5 consecutive days of training for weeks 5–8 in subject A and for each week of intensive training in subject B. Subsequent gain in performance was observed after the 2-day rest (Fig. 3). The greater reductions in performance were observed in both subjects for weeks 8 and 9. In addition, the subjects did not completely recover their best performance of week 9 after the 2-day rest between weeks 9 and 10. This incomplete recovery corresponded to the breaking in the general progression of the subjects’ fitness and could arise from short-term overtraining (14). In addition, the performance changed with some irregularities in the time course of the subsequent period of reduced training (weeks 11–14). Despite the low degree of exercise stress, a rather large
decrease in performance was observed for week 13 in subject A and for weeks 12 and 14 in subject B.

**TIME-IN Variant MODEL**

Let \( p(t) \) and \( w(t) \) be the time functions of performance and training, respectively; \( p(t) \) and \( w(t) \) are mathematically related as

\[
p(t) = p* + w(t) \ast g(t)
\]

where \( p* \) is an additive term that depends on the initial training status of the subject; \( g(t) \) is the impulse function defined in Eq. 1, and \( \ast \) denotes the product of convolution.

The definition of the convolution product leads to

\[
p(t) = p* + \int_0^t w(t-t') g(t') \, dt'
\]

The subjects trained regularly for the 4 mo preceding the experiment. This training should be taken into account. Because the model applied to subjects undergoing moderate training showed low-amplitude factor and time constant for the negative function (8), only the positive influence of the pretraining was considered in the estimation of the subjects’ initial status in the present study. The performance at the beginning of the experiment was thus the sum of the basic level of performance \( p* \) and a part due to past training that was likely to decrease with the time constant \( \tau_1 \). The discretization of Eq. 4 results in estimation of the model performance on day \( n (\hat{p}_n) \), from the successive training loads \( w_i \), with \( i \) varying from 1 to \( n-1 \)

\[
\hat{p}_n = p* + k_1 w(0) e^{-n \tau_1} + k_1 \sum_{i=1}^{n-1} w_i e^{-(n-i) \tau_1}
\]

\[
- k_2 \sum_{i=1}^{n-1} w_i e^{-(n-i) \tau_2}
\]

\( w(0) \) represents the accumulated training before the experiment. The training before the experiment was 100–200 units/wk. If a daily training of 20 units and a \( \tau_1 \) value of 50 days is assumed, the accumulated training function will plateau at 1,000 arbitrary units, given by \( w(0) = 20/[1 - \exp(-1/50)] \). This value of 1,000 units was thus chosen for \( w(0) \). The \( p* \) value was fixed at 80% of the value of performance at the beginning of the experiment, \( p* = 0.8 p_1 \). This basic level would correspond to the subjects’ performance after a few months of detraining.

The model parameters were considered to be constant throughout the 14-wk period. They were determined by fitting the model performances to the actual performances by the least square method, i.e., by minimizing the residual sum of squares (RSS) between modeled and actual performances

\[
RSS = \sum_N (\hat{p}_n - p_n)^2
\]

where \( n \) takes the \( N \) values corresponding to the days of measurement of the actual performance. Successive minimization of RSS with a grid of values for \( \tau_1 \) of

![Fig. 2. Best performance and training dose for each week of training.](image1)

![Fig. 3. Change in performance with 5-day training and 2-day rest for each week of intensive training.](image2)

![Fig. 4. Change in performance with 5-day training and 2-day rest for each week of intensive training.](image3)
30–60 days and \( \tau_2 \) of 1–20 days gave the total set of model parameters. The choice of these ranges of values for the time constants was guided by their estimates in the previous studies.

The model with time-invariant parameters accounted for almost 70% of the total variations in performance in both subjects (Table 1). The time constant of the positive component was 60 days for both subjects, whereas the time constant for the negative component was 4 days in subject A and 6 days in subject B. The corresponding values for \( \tau_1 \) were 6 and 9 days, respectively, in subjects A and B (Table 1). The estimates for \( \tau_1 \) reached the upper limit of the admissible values for both subjects. However, higher values for \( \tau_1 \) had little effect on the goodness of fit. Variations in \( \tau_1 \) have a minor effect on the time course of the performance response to a training impulse, compared with changes caused by variations in \( k_1, k_2, \) and \( \tau_2 \) (13).

The variations in performance during the second half of the experiment were not well described by the time-invariant model. The residual error after model fitting was rather large from week 8 to the end of the experiment (Fig. 4). The model accounted neither for the best performances during this period nor for the

<table>
<thead>
<tr>
<th>Subject</th>
<th>Model</th>
<th>( k_1 )</th>
<th>( k_2 )</th>
<th>( \tau_1 )</th>
<th>( \tau_2 )</th>
<th>( r^2 )</th>
<th>SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>A (n = 48)</td>
<td>Time-invariant</td>
<td>0.0021</td>
<td>0.0078</td>
<td>60</td>
<td>4</td>
<td>0.682</td>
<td>0.82</td>
</tr>
<tr>
<td></td>
<td>Time-varying Mean</td>
<td>0.0025</td>
<td>0.0082</td>
<td>56</td>
<td>5</td>
<td>0.875</td>
<td>0.52</td>
</tr>
<tr>
<td></td>
<td>CV (%)</td>
<td>62</td>
<td>78</td>
<td>16</td>
<td>108</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B (n = 49)</td>
<td>Time-invariant</td>
<td>0.0019</td>
<td>0.0073</td>
<td>60</td>
<td>6</td>
<td>0.666</td>
<td>0.87</td>
</tr>
<tr>
<td></td>
<td>Time-varying Mean</td>
<td>0.0022</td>
<td>0.0090</td>
<td>52</td>
<td>8</td>
<td>0.879</td>
<td>0.53</td>
</tr>
<tr>
<td></td>
<td>CV (%)</td>
<td>35</td>
<td>50</td>
<td>20</td>
<td>70</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

CV, coefficient of variation; \( k_1 \) and \( k_2 \), gain terms; \( \tau_1 \) and \( \tau_2 \), time constants.
large decreases in performance during weeks 8 and 9. Therefore, the model with parameters invariant over time failed to describe adequately the changes in performance when the fatigue would be accrued with intensive training.

TIME-VARYING MODEL

Equation 5 was fitted by a recursive least squares algorithm with a use of an exponential window (16). In such a process, the model parameters were estimated each time data were collected. The parameters at a given time were obtained from the previous and present data. The effect of present data was artificially emphasized by exponentially weighting past data values. On day \( n \), the parameters were fitted by minimizing the following recursive function

\[ S_n = S_{n-1} + (\hat{p}_n - p_n)^2 \]

where \( 0 < \alpha < 1 \). A small value of \( \alpha \) allowed rapid changes in model parameters, but they were sensitive to noise. A large value of \( \alpha \) limited the ability of the process to follow variations in parameters but reduced their sensitivity to noise. On day \( n \), \( k_1 \) and \( k_2 \) were computed for a grid of values for \( \tau_1 \) and \( \tau_2 \), identical to those used for the time-invariant model. Successive minimization of \( S_n \) gave the total set of parameters on day \( n \). However, the model parameters for day 1 were fixed. The negative function was considered to be zero (\( k_2 = 0 \)), and \( \tau_1 \) was fixed at 50 days. The \( k_1 \) value for day 1 was then computed by \( k_1 = 0.2 p_1/1,000 e^{-1/50} \). For instance, \( k_1 \) would be close to 0.001 arbitrary unit if the \( p_1 \) was 5 min.

The freedom given to the parameters to vary over time induce a reduction in the residuals after model fitting (Fig. 4). A better fit as a result of parameters free to vary over time was expected. Nevertheless, the changes in model parameters could arise from artifacts in model procedures. The value of \( \alpha \) was chosen in this study to give an SE close to 0.5 min to limit the influence of noise in parameter variations. The value

Fig. 5. Variations in parameter estimates and derived variables (mean values per week) when using time-varying model. \( \tau_1 \) and \( \tau_2 \), time constants; \( k_1 \) and \( k_2 \), gain terms; \( t_n \), time needed after impulse training stimulus for the effects of fatigue to be dissipated; \( p_2 \), maximal gain in performance due to 1 unit of training.
retained for $\alpha$ was 0.9 for both subjects, giving an SE of 0.52 for subject A and 0.53 for subject B (Table 1). Whereas this choice was arbitrary, the signification of the changes in model parameters over time needs to be examined.

**VARIATIONS OVER TIME IN MODEL PARAMETERS**

The mean estimates of the parameters fitted with the recursive algorithm were close to the values for the time-invariant model. Rather large variations, ascribed to the coefficient of variation (CV%), Table 1), were observed for $k_2$ and $\tau_2$ in both subjects and for $k_1$ in subject A.

Additionally to $t_n$, estimated by using Eq. 2, the time needed to reach maximal performance after an impulse training stimulus ($t_0$) was estimated as follows (13, 20)

$$t_0 = \frac{\tau_2}{\tau_1 - \tau_2} \ln \left( \frac{t_2}{t_1} \right)$$

(7)

The maximal gain in performance due to 1 unit of training was labeled $p_0$ and estimated as

$$p_0 = k_1 e^{-t_0/t_1} - k_2 e^{-t_0/t_2}$$

When the model parameters and the derived variables are plotted as a function of time (Fig. 5), it appears that the amplitude factors $k_1$ and $k_2$ and the more composite variable $p_0$ increased in the time course of the experiment. However, the time constants $\tau_1$ and $\tau_2$ and the derived value $t_0$ did not change in any simple way.

To test the consistency of the variations over time of the parameter estimates, their variability has to be compared with some variability in the data. The aim of the model is to distinguish short-term fatigue to long-term adaptation. Because the progression in performance was not steady over the time course of the experiment, the best performance reached during each week could be used as an index of the subjects’ adaptability. On the other hand, the change in performance with the 5 successive days of intensive training would be an indicator of the magnitude of the fatigue resulting from exercise stress. The model parameters and derived variables were compared with both the best performance per week and the change in performance with the 5 consecutive days of each week of intensive training by using linear regression. The amplitude factor $k_1$ was correlated to the best performance per week: $R = 0.71 (P < 0.01, n = 14)$ in subject A and $R = 0.61 (P < 0.05, n = 14)$ in subject B. Figure 6 illustrates the similar results obtained for $p_0$: $R = 0.86 (P < 0.001)$ in subject A and $R = 0.71 (P < 0.01)$ in subject B. In addition, the variation in performance with the 5 consecutive days of intensive training was correlated with $k_2$: $R = 0.80 (P < 0.05, n = 7)$ in subject A and $R = 0.68 (P = 0.06, n = 8)$ in subject B and with the difference $k_2 - k_1$: $R = 0.84 (P < 0.05)$ in subject A and $R = 0.72 (P < 0.05)$ in subject B (Fig. 7). Week 3 was not considered for subject A, since he did not complete the 5-day training. The time constants, $t_n$, and $t_0$ failed to show such correlations.

The breaking in the performance progression at the end of the second period of intensive training corresponded to an incomplete recovery of the performance with the 2-day rest between the weeks 9 and 10. To assess how the variations in model parameters accounted for this feature, the time impulse response to a training amount of 100 units was computed in both subjects for weeks 9 and 10 (Fig. 8). The time needed to recover performance after a training impulse was greater in week 10 (14 days in subject A and 10 days in subject B) than in week 9 (5 days in both subjects). In addition, $\tau_2$ was greater in week 10 (17 days in subject A and 11 days in subject B) than in week 9 (2 days in subject A and 5 days in subject B).

The greater values for $\tau_2$ for subject B during the first part of the experiment are in line with the greater decreases in performance observed for this subject with intensive training on weeks 1-4 (Fig. 3). However, it is difficult to assess whether the variations in model parameters during the second period of reduced training would reflect the persistence of the effects of the preceding intensive training or would be artifacts arising from the irregularities in the performance evolution during the last weeks of the experiment.

![Fig. 6. Relationship between $p_0$ and best performance reached during each week of training.](image336x87-540x244)

![Fig. 7. Relationship between difference $k_1 - k_2$ and change in performance with the 5-day training for each week of intensive training.](image335x589-to-539x727)
The conclusions that can be drawn are as follows: 1) the variations over time in the amplitude factors $k_1$ and $p_g$ were in keeping with the nonsteady week-to-week improvement of the performance; 2) the amplitude factor $k_2$ and the difference between $k_1$ and $k_2$ changed over time in line with the response to the 5 consecutive days of intensive training; 3) the increase in the time constant $\tau_2$ at the end of the second period of intensive training was in accordance with the incomplete recovery of the performance observed at this time; 4) however, artifacts in parameter variations could arise from unevenness in performance during the last weeks of the study.

In conclusion, the purpose of the present study was to assess whether a time-varying systems model would provide more information on the adaptation to training than the model with time-invariant parameters. The data gathered to examine the usefulness of the proposed time-varying model showed a nonsteady improvement in performance with a critical period where the subjects had great difficulty recovering from exercise. These particular features of the data were described by the variations over time in model parameters: changes in $k_1$ and $p_g$ as indicators of the increase in the benefit of training with repeated exercise, and changes in $k_2$ and $\tau_2$ that were in line with apparent modifications in the magnitude and the time course of the long-term fatigue resulting from the intensive training. These observations are partly in accordance with the data in the literature concerning the time-invariant model. The increase in the negative influence of training is in keeping with the greater values for $k_2$ and $\tau_2$ observed for a greater intensity of training (see basic framework). In contrast, the increase in the amplitude factor $k_1$ with intensive training is less well established. In the present study, the training doses were estimated from exercise intensity, taking into consideration the long-term improvement in subjects' fitness. However, since the negative influence of training affected performance, the input signal corresponding to a given training session also could be enhanced, yielding to greater adaptation and fatigue. A more precise assessment of the training doses thus will be necessary in future investigations to address adequately this issue.

The changes over time in model parameters would not arise generally from noise in data. However, these variations cannot be directly interpreted as modifications in the underlying physiological mechanisms. The model with time-invariant parameters considers the adaptations to physical training as an addition over time of the effects of each training impulse. In contrast, parameters free to vary over time allow the model to better describe the complexity of the cumulated effects of the training by considering that the influence over time of a given training impulse could be dependent on the previous and later training doses. The observed changes in model parameters in the time course of training with varied regimens could thus arise from the model structure rather than from physiological alterations. Nevertheless, the emphasis of the apparent amplitude factors and time constants is to describe the observed behavior of subjects undergoing physical training. The present findings showed that the proposed model with parameters varying over time can be a useful tool in studies and may provide a way of studying the mechanisms underlying adaptations to physical training. The particular features of the present data concerning increase in both adaptability and long-term fatigue with intensive training deserve further investigations to assess their reproducibility in a larger number of subjects and possible correspondence with physiological entities.

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