Predicting metabolic rate across walking speed: one fit for all body sizes

Peter G. Weyand,1 Bethany R. Smith,2 Nicole S. Schultz,1 Lindsay W. Ludlow,1 Maurice R. Puyau,1 and Nancy F. Butte3

1Southern Methodist University, Locomotor Performance Laboratory, Department of Applied Physiology and Wellness, Dallas, Texas; 2St. Luke’s Episcopal Hospital, Houston, Texas; and 3USDA/ARS Children’s Nutrition Research Center, Department of Pediatrics, Baylor College of Medicine, Houston, Texas

Submitted 5 November 2012; accepted in final form 2 August 2013

THE PREMISE THAT THE METABOLIC energy walking requires is set by gait mechanical demands is universally accepted. However, in more than a century of experimentation, a broad predictive relationship that directly links the two has not come forth. The absence of a relationship has not resulted from any lack of scientific interest in the topic; hundreds of studies have investigated walking energetics for humans and other terrestrial species. Indeed, one could easily conclude from the exhaustive literature on human walking that the relationship is too complex to be described in simple quantitative terms (8, 23, 32, 42, 49).

However, the primary findings from the comparative work on terrestrial locomotion offer a more promising perspective (27, 31, 43, 47, 48, 58). These classical studies identified an economy of scale for locomotor energy expenditure and provided a mechanistic explanation for its basis. Early investigations (47) established that the energy expended to transport 1 kg of tissue 1 m (E\text{trans}, \text{energy-kg}^{-1}\text{m}^{-1}) varied with the body mass (M_b) of the animal to the negative one-third power (E\text{trans} \propto M_b^{-0.32}). Subsequent experiments demonstrated that, at the equivalent speeds of different-sized quadrupeds, such as trot-gallop transition speed, the mass-specific energy expended per stride was nearly invariant (27, 45). In this latter case, the superior economy of the larger animals was explained by their greater limb (L\text{leg}) and stride lengths (L\text{str}). These results implied an inverse relationship between transport costs and L\text{leg} values (E\text{trans} \propto L\text{leg}^{-1.0}) at equivalent speeds.

Although a body-size dependency of human locomotor economy has been apparent since the middle of the last century (5), the experimental approaches developed by comparative biologists have been applied only limitedly to humans. The few investigations of the body-size basis of human walking economy that have been undertaken have reported trends similar to those appearing in the comparative literature (21, 35, 54). The lone human analysis quantifying the scaling of walking transport costs with M_b at mechanically equivalent speeds (54) conformed closely (E\text{trans} \propto M_b^{-0.29}) to the classic comparative results. This study also found that the mass-specific energy expended per stride at a single equivalent walking speed did not vary between small and large human walkers. Thus, at the single standardized speed examined, the walking transport costs of humans were inversely related to L\text{leg} (E\text{trans} \propto L\text{leg}^{-0.95}).

The apparent conformation of human locomotor mechanics and metabolic energetics to consistent patterns, and the overarching importance of walking energy expenditure for human health, function, and physiological status, begs the question of why a mechanics-based relationship for predicting energy expenditure has not emerged. A primary reason appears to be a lack of integration between the experimental approaches thus far employed. Those studies focused on developing generalized equations for predictive purposes have typically relied more heavily on post hoc statistical analyses than a priori mechanistic formulations (7, 16, 17, 37, 38, 52). Most have used regression analyses with limited incorporation of established knowledge or theory. For example, the Pandolf et al. (38) and American College of Sports Medicine (ACSM) (3) equations, which are the current predictive standards, were both derived empirically and without incorporating the influence of gait mechanics or limb lengths. In contrast, the more basic studies that have incorporated the germane physiological and mechanical relationships a priori have generally not been applied for predictive purposes (19, 21, 35, 54).

Our objective here was to develop a generalized predictive equation for human walking economy that draws on compar-
ative traditions to more fully incorporate the influence of body size. For this purpose, we formulated a simple, mechanistic, whole-body model of walking metabolism and gait mechanics that includes three basic variables: height, weight, and walking speed. We acquired metabolic data across a broad range of level walking speeds for human subjects who spanned a twofold range of statures and a sevenfold range of body mass values to test two hypotheses. First, we hypothesized that the best-fit, empirical descriptions provided by our model would be similar when derived from short, moderately short, moderately tall, and tall human walkers. Second, we hypothesized that our basic model would predict metabolic rates during level walking with appreciably greater accuracy than either of the two leading current standards, the ACSM and Pandolf et al. equations.

METHODS

The Height-Weight-Speed Model

Our three-component model of walking metabolism is illustrated in Fig. 1. Rates of energy expenditure are illustrated as a function of walking speed, with the former expressed in units of oxygen uptake per physiological convention. Mass-specific rates of oxygen uptake typical for a tall adult appear on the left Y-axis, while metabolic rates, expressed in multiples of the body’s resting rate (METs), appear on the right Y-axis. The standardized values, theorized to apply to an individual of any height and weight, have been included to illustrate the model’s postulated applicability across a broad continuum of human body sizes. The model partitions gross walking metabolic rates into three components: 1) resting metabolism, 2) minimum walking metabolism, and 3) speed-dependent walking metabolism. The scientific rationale for the model follows.

Resting metabolic rate. The model’s first component is the minimum metabolic rate needed to supply all the body’s tissues at rest, or resting metabolic rate (RMR). This component, in contrast to the other two in the model, can be directly measured under standardized conditions. For modeling purposes, we have assumed that RMRs accurately represent the minimum metabolic rate needed to sustain the body’s tissues at rest and during exercise, and that this quantity is constant across different walking speeds.

Minimum walking metabolic rate. The model’s second component is the minimum metabolic rate needed, above the body’s resting rate, for walking at any speed. We have termed this component the minimum walking metabolic rate (MWMR). The primary contributors to the MWMR are the metabolic costs incurred to maintain an upright posture and support the body’s weight against gravity in a walking posture (10, 46). Secondary contributors include the slight elevations in cardiac and pulmonary muscle activity needed to support increased pulmonary oxygen uptake and cardiovascular transport (46), and perhaps other factors. For modeling purposes, we have assumed that the MWMR, like RMR, remains constant across walking speeds.

Speed-dependent walking metabolic rate. The model’s third component is that portion of the gross walking metabolic rate attributable to walking speed. The primary contributor to this third model component is the increased metabolic cost of supporting the body’s weight against gravity at faster walking speeds. This cost increases with speed as muscle fibers with greater rates of ATP utilization are recruited to support the body’s weight during progressively shorter periods of foot-ground force application at faster speeds (28, 30, 40, 46). Secondary contributors include performing the limited mechanical work per step required to lift and accelerate the body’s mass, and the relatively small metabolic cost of swinging the limbs at faster walking speeds. Indirect evidence suggests that the two latter factors, although relatively small, do contribute to the increased slope of the metabolic rate-walking speed relationship across the fastest walking speeds (4, 14, 15, 18).

Formulaic basis of the model. Of the three basic predictors in the Height-Weight-Speed model, the most straightforward influence is that of the total weight supported against gravity, which is typically the weight of the body. This direct influence is present in experimental results from load carriage studies (9, 26, 38), longitudinal studies involving weight loss (6, 24, 29), cross-sectional studies comparing obese and nonobese individuals (14, 15, 33), mechanical explanations of locomotor metabolism (10, 28, 31, 40, 45, 46), and in the form in which $M_b$ has been widely incorporated into existing predictive equations (3, 16, 17, 19, 38, 51, 56, 57). All of the aforementioned experimental and predictive results are consistent with the conclusion that, when the other factors (height and walking speed) are held constant, a 1:1 relationship exists between the body weight supported and the metabolic energy walking requires. Hence, the widespread convention of expressing the metabolic rates observed during locomotion and other weight-bearing exercise in mass-specific terms enjoys extensive experimental support. Accordingly, we have incorporated $M_b$ directly into the denominator of all the metabolic terms in our height-weight-speed model as follows:

$$\dot{V}_{O_2}^{gross} = \frac{\dot{V}_{O_2}^{rest} + C_1 \cdot \dot{V}_{O_2}^{rest} + (C_2 \cdot V^{exp}) \cdot Ht^{-1}}{M_b}$$

where $\dot{V}_{O_2}^{gross}$ is the body’s total, or gross volume rate of oxygen uptake, $\dot{V}_{O_2}^{rest}$ is the body’s resting rate of oxygen uptake, $C_1$ is a coefficient describing the minimum walking rate of oxygen uptake as a multiple of the resting rate, $C_2$ is the coefficient that describes speed-dependent increases in the rate of oxygen uptake as a function of the velocity of walking, $V$, raised to the exponent, $exp$, divided by the height, $Ht$, or stature of the individual. Hence, the sum of the model’s second and third metabolic components represents the metabolic rate attributable to walking ($\dot{V}_{O_2}^{walk}$). To be consistent with prior literature, all of the terms in Eq. 1 above are expressed in mass-specific units of oxygen uptake of $ml O_2$ $kg^{-1}$ $min^{-1}$, where $Ht$ is in m, and $V$ is in $m/s$. Per our scientific objectives and both Fig. 1
and Eq. 1, the term metabolic rate is used to refer to mass-specific rates of oxygen uptake throughout the paper.

The quantitative form of the first of our model’s three metabolic components (\(V\dot{O}_2\) rest, Eq. 1), the body’s RMR, is largely self-explanatory, because RMRs are a standard and universally accepted measure. The second model component, the body’s MWMR, incurred predominantly by support and postural requirements, was assumed to be constant across speed at a fixed multiple of the body’s RMR (\(C_1 \cdot V\dot{O}_2\) rest, Eq. 1) largely on the basis of prior results (19, 21, 37, 54, 56). The most appropriate form for the model’s third component, speed-dependent walking metabolic rate (SDWMR), is more difficult because the speed-induced increases in walking metabolic rates depend on stature (54). We postulated that the speed-dependent portion of walking metabolic rates would be an exponential function of velocity and an inverse function of height (\(V^{3.0} - H^{-1}\)) for the following reasons. First, both mechanics-based approaches and correlational modeling have been consistent in the finding that the increases in walking metabolic rates that occur with speed can be reasonably well described as a function of the velocity of walking squared (\(V^2\)) (19, 38, 51, 56). Second, among individuals who differ in body size, metabolic rate increases that occur with increases in walking speed are systematically greater in shorter vs. taller individuals, and, therefore, inversely related to stature (21, 54). Hence, the model’s third metabolic rate term takes the form of a coefficient times walking velocity raised to an exponent divided by height [\(C_3 \cdot V^{3.0} - H^{-1}\), Eq. 1]. In those instances in which \(exp\) has the theorized value of 2.0, this \(V^2 - H^{-1}\) term reduces to units of meters per second squared.

Our model incorporates an existing solution for identifying speeds that are mechanistically equivalent for individuals who differ in stature. This solution is derived from the principle of dynamic similarity, and has, in prior literature (1, 2, 54), taken the form of the Froude number: \(U = \frac{V^2}{g \cdot \text{Lleg}}\) where \(U\) is equivalent speed, and \(g\) is gravitational acceleration. Our prior result at a single equivalent speed indicated that different-sized human walkers do indeed walk in a dynamically similar manner (54), which by definition entails stride lengths, times, and forces being related to the body’s linear dimensions by a constant across the full continuum of body sizes (1, 2). In addition, we found that the energy cost per kilogram per stride for shorter and taller individuals at the one equivalent speed examined did not vary. If our prior metabolic result from one equivalent speed generalizes to other equivalent speeds, then a single term that includes the speed-induced increases in walking metabolic rates depend on stature (54). We postulated that the speed-dependent portion of walking metabolic rates would be an exponential function of velocity and an inverse function of height (\(V^{3.0} - H^{-1}\)) for the following reasons. First, both mechanics-based approaches and correlational modeling have been consistent in the finding that the increases in walking metabolic rates that occur with speed can be reasonably well described as a function of the velocity of walking squared (\(V^2\)) (19, 38, 51, 56). Second, among individuals who differ in body size, metabolic rate increases that occur with increases in walking speed are systematically greater in shorter vs. taller individuals, and, therefore, inversely related to stature (21, 54). Hence, the model’s third metabolic rate term takes the form of a coefficient times walking velocity raised to an exponent divided by height [\(C_3 \cdot V^{3.0} - H^{-1}\), Eq. 1]. In those instances in which \(exp\) has the theorized value of 2.0, this \(V^2 - H^{-1}\) term reduces to units of meters per second squared.

Our model incorporates an existing solution for identifying speeds that are mechanistically equivalent for individuals who differ in stature. This solution is derived from the principle of dynamic similarity, and has, in prior literature (1, 2, 54), taken the form of the Froude number: \(U = \frac{V^2}{g \cdot \text{Lleg}}\) where \(U\) is equivalent speed, and \(g\) is gravitational acceleration. Our prior result at a single equivalent speed indicated that different-sized human walkers do indeed walk in a dynamically similar manner (54), which by definition entails stride lengths, times, and forces being related to the body’s linear dimensions by a constant across the full continuum of body sizes (1, 2). In addition, we found that the energy cost per kilogram per stride for shorter and taller individuals at the one equivalent speed examined did not vary. If our prior metabolic result from one equivalent speed generalizes to other equivalent speeds, then a single term that includes the \(V^2\) divided by the linear dimensions of the body should accurately describe the speed-dependent metabolic rates of different individuals, regardless of their height. Here, for simplicity and ease of use, we used a Froude number analog that replaced \(\text{Lleg}\) with body length (i.e., height) and dropped the gravitational acceleration term to become: \(V^2 - H^{-1}\).

Our equivalent speed term for this third model component led us to two specific predictions. First, speed-dependent increases in mass-specific metabolic rates should be linear when expressed in relation to the \(V^2\). Second, the differences in how rapidly metabolic rates increase as a function of speed for shorter vs. taller individuals should be an inverse function of both \(\text{Lleg}\) and height. Neither sex nor age was included in the model, because both mechanical theory and prior empirical results (54) indicate these variables do not influence walking economy independently of height, weight, and speed in healthy individuals under 50 yr of age.

**Experimental Protocol and Measurements**

**Subjects.** Two strategies were employed to maximize the range of body sizes and walking metabolic rates obtained. First, we recruited human subjects who spanned a wide range of heights and weights. Second, we tested subjects across a nearly fivefold range of walking speeds from 0.4 to 1.9 m/s. By recruiting children as young as 5 yr of age and enrolling a number of individuals whose stature exceeded 2.0 m (>6’6’’), we obtained a nearly twofold range of statures (1.07–2.11 m) and sevenfold range of \(M_v\) values (15.9–112.8 kg) in our subject pool. We ultimately tested a total of 78 subjects, 45 males and 33 females, between the ages of 5 and 48 yr. Per the protocols approved by the local Institutional Review Boards, adults provided written, informed consent, while children provided written assent accompanied by the written consent of a parent or legal guardian. Subjects were healthy and generally free of obesity as only 4 of the 78 subjects had BMI values >30 kg/m\(^2\). Limited data from 48 of the 78 subjects were reported in a prior study (54). Height and weight were measured with a stadiometer and platform scale, accurate to the nearest 0.001 m and 0.1 kg, respectively. \(\text{Lleg}\) values were measured by palpating the hip joint axis of rotation during standing and slow swinging of the limb in the sagittal plane.

**Treadmill testing protocol.** Subjects were asked to walk on a level treadmill at constant speeds of 0.4, 0.7, 1.0, 1.3, 1.6, and 1.9 m/s. Trials were administered in a staggered-speed fashion, with slower speeds being completed earlier and faster speeds being completed later in the protocol. Some of the shortest subjects did not complete trials at the fastest one or two protocol speeds because they could not do so without running. Hence, the exact sequence of trial speeds necessarily varied across subjects. Each trial lasted long enough to obtain a 2-min, steady-state rate of oxygen uptake, typically from 4 and 6 min. Subjects were given a 5- to 10-min break after completion of the protocol before repeating all trial speeds a second time.

**Metabolic measures.** A computerized metabolic system (Parvo Medics TrueOne 2400, Sandy, UT) was used to measure rates of metabolic energy expenditure as assessed from measured rates of oxygen uptake (11). Samples of expired gases during steady-state treadmill walking were taken and analyzed for CO\(_2\) and O\(_2\) fractions using infrared and paramagnetic gas analyzers, respectively. Respiratory gases were collected using a one-way breathing valve that directed expired air through a pneumotach into a mixing chamber before analysis. For each speed, rates of oxygen uptake were averaged over a 2-min, steady-state period, and the steady-state values from the two protocol repetitions were averaged for subsequent data analysis (54). Calibration was performed using a 3-liter syringe to direct air through the system at volume flow rates similar to ventilation rates encountered during testing. A two-point calibration procedure was used to calibrate the gas analyzers using room air and a gas cylinder containing known concentrations of O\(_2\) and CO\(_2\) in the physiological range for expired gases. The TrueOne system was also validated in the range of rates of oxygen uptake from 0.3 to 1.01 l/min via simulations using precision blended N\(_2\)-CO\(_2\) mixtures, according to the infusion technique described by Moon et al. (36). The agreement between the rates of oxygen uptake measured by the TrueOne system across 15 infusion tests spanning these simulated rates of oxygen uptake was <3.0%, as previously reported (54).

In previous work, we have converted measurements of oxygen uptake to metabolic rates or rates of energy expenditure using an energetic equivalent of oxygen of 20.1 J/ml O\(_2\) (11). However, given the largely applied objective of the present study and existing literature conventions, here we report all results as rates of oxygen uptake (ml O\(_2\)-kg\(^{-1}\)-min\(^{-1}\)) without conversion to true units of energy for ease of interpretation.

**Kinematic measures.** Walking kinematics were obtained using a 30-Hz video (Sony model DCR-TRV19, 30 Hz). Stride times (\(t_{str}\)) were determined by counting the frames of 25 sequential contact periods of the same foot. Stride time was defined as the time between successive footfalls of the same foot. Stride frequency, the inverse of stride time, was determined to quantify the energy expended per kilogram-stride\(^{-1}\) as previously shown (Ref. 54, where energy expended kg\(^{-1}\)-stride\(^{-1}\) = \(V\dot{O}_2\) walk/\(t_{str}\)).

**Data Treatment, Analysis, and Hypothesis Tests**

**Data management.** Two considerations guided our model from scientific formulation to hypothesis testing: predictive accuracy and ease of use. We maximized predictive accuracy by allowing three
numerical values in our model to be those that provided the best fit to the data across the broad ranges of height, weight, walking speeds, and metabolic rates in our data set. These three values were as follows: the coefficient describing the MWMRs ($C_1$), the coefficient describing speed-dependent increases in metabolic rates ($C_2$), and the exponent (exp) describing speed-dependent increases in metabolic rates (Eq. 1). The optimization function in Excel (Ref. 25; Microsoft Excel Solver, Excel 2010 version) was used for this purpose because this tool has the ability to identify the three aforementioned values while other model inputs (height, weight, estimated RMR, and walking velocity) were fixed at their known values. Thus the values of $C_1$, $C_2$, and exp that we report were those that allowed our model to best fit (i.e., maximized the explained variance) the experimental group metabolic rate data using a linear model. We used estimated rather than measured RMRs ($V_{O_2}$rest) because the majority of potential users do not know, and cannot acquire, their measured resting values. The equations of Schofield et al. (44) that we used for this purpose have been extensively validated and are typically accurate to within 0.5 ml O$_2$ kg$^{-1}$ min$^{-1}$ or less (22, 30, 39, 41, 50, 55). Hence, any error introduced by using estimated, rather than directly measured, RMR values is likely to constitute a very small fraction of gross metabolic rates during walking, in accordance with our laboratory’s prior findings (54).

**Hypothesis test one.** We tested our first hypothesis that the best-fit, empirical descriptions provided by our model would not differ for human walkers of different statures in the following manner. We divided our sample of 78 total subjects into four groups exclusively on the basis of height to obtain: a short group (A), moderately short group (B), moderately tall group (C), and a tall group (D). We then extracted four subjects from the midrange of statures within each of these groups to form a reference group with subjects who, with respect to stature, were representative of each of the four groups from which they were drawn. Four fully independent best-fit metabolic rate-speed equations were then derived in the form of the height-weight-speed model (Eq. 1) using the walking metabolic rate data obtained from the subjects in each of the four respective stature groups. This process produced four best-fit equations with values of $C_1$, $C_2$, and exp derived independently to optimize the model fit to subjects of different statures. The number and characteristics of the subjects from the four stature and one reference group appear in Table 1.

Once derived, the respective equations from each of the four stature groups were then used to generate predicted walking metabolic rate values at each of the six walking speeds in our protocol. Our expectation was that, due to the manner in which our model incorporates height, weight, and walking speed, the values predicted at each speed would be similar, despite being derived on groups that differed in stature and body mass. The subject-specific model inputs required to generate the predictions: height, weight, and estimated RMR, were in each case provided by the reference group subjects to hold the input variables fully constant across the four predictive equations. We specifically evaluated whether the metabolic rate predictions generated by the four stature group equations differed significantly by using a two-way ANOVA ($\alpha < 0.05$) that tested for the main effects of stature group and walking speed.

**Hypothesis test two.** We tested our second hypothesis that our basic model would, after empirical refinement, predict walking metabolic rates during level walking appreciably more accurately than either the ACSM or Pandolf et al. equations, as follows. We split our sample total of 78 subjects into experimental and validation groups of 39 subjects each. One subject from each of 39 pairs of stature-matched individuals was randomly assigned to the experimental group, while the remaining member of the pair was assigned to the validation group. The heights, weights, and sexes of the experimental and validation group for hypothesis test two also appear in Table 1. A best-fit equation in the form of Eq. 1 was developed from the measured metabolic rates of the experimental group subjects. As with hypothesis test one, the coefficients $C_1$ and $C_2$ and exponent exp were allowed to vary to provide the best linear fit to the metabolic rate data of the experimental group. The best-fit equation derived on the experimental group was then used to predict the walking metabolic rate measured for the subjects in the validation group at each of the six walking speeds. The overall agreement between measured values and those predicted by the model was assessed from the proportion of total variance in walking metabolic rates accounted for ($R^2$) via linear regression and deviation from the line of identity. The accuracy of the individual predictions was assessed using the standard error of estimate (SEE). We hypothesized that the predictive error of the Height-Weight-Speed model would be less than one-half that of the ACSM and Pandolf et al. equations for the group of subjects and speeds tested here.

In addition to the ACSM and Pandolf equations, and to provide a more comprehensive perspective on the relative predictive accuracy of our Height-Weight-Speed model, we also generated predictions from three other prominent predictive equations in the literature: Cotes and Meade (19), van der Walt and Wyndham (51), and Workman and Armstrong (56).

**RESULTS**

**Part I: Representative Subject Data to Illustrate the Height-Weight-Speed Model**

**Metabolic rates: gross, net walking, and speed-dependent rates vs. absolute speed.** The gross metabolic rates of four representative subjects whose heights (A: 1.15, B: 1.47, C: 1.78, and D: 2.06 m) spanned a 1.8-fold range appear in Fig. 2. The taller the individual, the lower the gross metabolic rates were at any given speed (Fig. 2A). Similarly, speed-induced increases in these rates also tended to be smaller for taller subjects. Differences between the shortest and tallest individuals in gross metabolic rates were in approximate proportion to their stature difference, being roughly twofold at each of the common speeds completed. A portion of the difference in gross rates was attributable to the greater RMR values estimated for the shorter individuals (Fig. 2D). When the first two metabolic components of the model, estimated RMR and MWMR (Fig. 1, Eq. 1), were sequentially subtracted to provide net walking metabolic rates (gross; RMR), and subsequently SDWMRs [gross: (RMR + MWMR)], the patterns observed for gross rates remained largely intact, but were substantially reduced (Fig. 2G). For each of the three expressions of walking metabolism, and each of the four individuals, metabolic rates increased in a curvilinear fashion with walking speed.

**Metabolic rates vs. equivalent walking speeds.** For the three expressions of walking metabolism, and each of the four

---

**Table 1. Physical characteristics of the subgroups for hypotheses 1 and 2**

<table>
<thead>
<tr>
<th>Hypothesis 1</th>
<th>n (Male)</th>
<th>Age, yr</th>
<th>$M_b$, kg</th>
<th>$L_b$, m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stature groups</td>
<td>1</td>
<td>15 (6)</td>
<td>$8.3 \pm 0.8$</td>
<td>$32.9 \pm 4.1$</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>15 (5)</td>
<td>$17.3 \pm 2.3$</td>
<td>$58.0 \pm 3.4$</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>16 (9)</td>
<td>$20.8 \pm 1.3$</td>
<td>$65.5 \pm 2.4$</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>16 (15)</td>
<td>$21.8 \pm 0.9$</td>
<td>$87.6 \pm 3.2$</td>
</tr>
<tr>
<td>Reference group</td>
<td>16 (10)</td>
<td>$19.6 \pm 2.9$</td>
<td>$63.5 \pm 4.8$</td>
<td>$1.62 \pm 0.05$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Hypothesis 2</th>
<th>n (Male)</th>
<th>Age, yr</th>
<th>$M_b$, kg</th>
<th>$L_b$, m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental group</td>
<td>39 (25)</td>
<td>$17.5 \pm 1.5$</td>
<td>$63.0 \pm 3.6$</td>
<td>$1.65 \pm 0.04$</td>
</tr>
<tr>
<td>Validation group</td>
<td>39 (22)</td>
<td>$17.9 \pm 1.3$</td>
<td>$60.8 \pm 3.6$</td>
<td>$1.62 \pm 0.04$</td>
</tr>
</tbody>
</table>

Values are means ± SE; n, no. of subjects. $M_b$, body mass; $L_b$, body length.
individuals, metabolic rates increased in a curvilinear fashion with walking speed, and, per above, these increases tended to be smaller for the taller individuals (Fig. 2, A, D, and G). When increases were plotted in relation to $V^2$, a slight curvilinear trend remained for each of the three metabolic expressions and for all four individuals, as did small individual differences in the rates at which metabolic rates increased (Fig. 2, B, E, and H). However, when the metabolic rate vs. speed squared relationship was subsequently divided by the stature of the individual, the small differences present before this stature standardization were essentially fully eliminated for all three of the metabolic expressions illustrated (Fig. 2, C, F, and I). Finally, when both RMR and MWMR were subtracted from gross metabolic rates, the slopes of the SDWMR of the four different individuals became essentially the same (Fig. 2I).

**Part III: Hypothesis Test Outcomes**

*Hypothesis test one: one fit from different body sizes?* The gross metabolic rates predicted by the four independent best-fit equations derived from the four stature groups formulated to test hypothesis one appear in Fig. 4. The derived values for the coefficients $C_1$, $C_2$, and exp for each of the four groups that were used to generate the predictions illustrated appear in Table 2, accompanied by the $R^2$ and SEE values for each of the respective within-group fits. The metabolic rates predicted...
using the four equations increased with speed (ANOVA, $P < 0.001$) from mean values at the slowest speed of 0.4 m/s of 7.5 ± 0.3 ml O$_2$·kg$^{-1}$·min$^{-1}$ to values of 19.0 ± 0.5 ml O$_2$·kg$^{-1}$·min$^{-1}$ at 1.9 m/s. In addition to similar values being predicted at the different protocol speeds, the overall pattern of increase for the metabolic rate-speed relationship was also similar across the four groups. The main effect of stature group on predicted metabolic rates was not significant (ANOVA, $P = 0.13$). The mean values predicted by the four equations for the reference group subjects agreed with one another to within an average of 5.2 ± 3.7% across the four intermediate speeds in the protocol ($n = 16$ comparisons).

**Hypothesis test two: a more accurate generalized equation?**

The relationship between the metabolic rates predicted by the ACSM and Pandolf et al. models and the metabolic rates measured for the 39 subjects in our validation group appear in Fig. 5, A and B. The proportion of the total variance of the walking metabolic rates measured for these subjects as provided by the $R^2$ value vs. the line of identity was 0.35 and 0.40 for the ACSM and Pandolf et al. equations, respectively, with corresponding SEE values of 3.35 and 3.23 ml O$_2$·kg$^{-1}$·min$^{-1}$, respectively. For the Height-Weight-Speed model derived here, the corresponding $R^2$ value for the proportion of the total variance accounted for was 0.90, while the SEE for the predicted values was 1.34 ml O$_2$·kg$^{-1}$·min$^{-1}$ (Fig. 5C). Hence, the error of individual prediction was roughly 2.5 times greater for ACSM and Pandolf et al. vs. the Height-Weight-Speed model. The relatively poorer predictive accuracy of the ACSM and Pandolf et al. equations resulted primarily from substantial underpredictions of the gross metabolic rates of the shorter subjects.

In addition to the generalized equation derived from the experimental group formed to test hypothesis two that appears in Table 2, we also derived a simplified version of the model equation with $C_1$ fixed at 1.00 and the exponent fixed at 2.00. The accuracy of the predictions provided by this simpler form of the equation differed little from the original equation reported in Table 2 ($R^2 = 0.87$; SEE = 1.53 ml O$_2$·kg$^{-1}$·min$^{-1}$).

The predictive accuracy of our Height-Weight-Speed model was essentially unchanged when $L_{eq}$ was used as a model predictor instead of height. In this case, the best-fit equation derived on the experimental group using $L_{eq}$ rather than height in the model [$\dot{V}O_2 \text{ gross} = \dot{V}O_2 \text{ rest} + 0.097 \cdot \dot{V}O_2 \text{ rest} + (2.56 \cdot V^{2.38}) \cdot L_{eq}^{-1}$] accounted for a slightly smaller proportion of the total variance with a slightly greater SEE ($R^2 = 0.87$; SEE = 1.49 ml O$_2$·kg$^{-1}$·min$^{-1}$) when predicting the walking metabolic rates of the validation-group subjects.

**DISCUSSION**

Basic and applied motivations led us to formulate and test a simple, whole body model that predicts the energy cost of level human walking from height, weight, and walking speed. The model is consistent with the body-size trends previously reported for human locomotor economy and physical principles that apply to the gait mechanics of terrestrial species from rodents to dinosaurs (1, 2). In keeping with our first hypothesis, the empirical, best-fit equations in the form of the model independently derived from four groups of subjects who differed in both stature and mass (Table 2) provided similar predictions. The mean values predicted from these four distinct model-derived equations for the same set of reference subjects agreed with one another to within an average of 5.2 ± 3.7% for the four intermediate speeds in our protocol (Fig. 4A) that encompass the range of speeds humans typically self-select in the field (13, 18). As hypothesized for our applied second hypothesis, the best-fit model equation derived on one-half of our subjects predicted the walking metabolic rates of the stature-matched remaining one-half to within 8.1 ± 6.7% on average ($R^2 = 0.90$; SEE = 1.34 ml O$_2$·kg$^{-1}$·min$^{-1}$) and appreciably more accurately than existing literature alternatives. Given the nearly twofold range of statures and sevenfold range of body masses of the subjects tested, these results indicate that the answer to the "one fit for all sizes" question.

**Table 2. Empirical derivations of model components**

<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>n (No. of subjects)</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>Exponent</th>
<th>$R^2$</th>
<th>SEE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stature 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>15</td>
<td>0.87</td>
<td>4.37</td>
<td>2.74</td>
<td>0.90</td>
<td>1.30</td>
</tr>
<tr>
<td>2</td>
<td>15</td>
<td>0.98</td>
<td>4.73</td>
<td>2.31</td>
<td>0.92</td>
<td>1.16</td>
</tr>
<tr>
<td>3</td>
<td>16</td>
<td>1.10</td>
<td>3.71</td>
<td>2.67</td>
<td>0.92</td>
<td>1.24</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
<td>1.29</td>
<td>3.83</td>
<td>2.57</td>
<td>0.90</td>
<td>1.20</td>
</tr>
<tr>
<td>Experimental</td>
<td>39</td>
<td>0.97</td>
<td>4.87</td>
<td>2.34</td>
<td>0.89</td>
<td>1.45</td>
</tr>
</tbody>
</table>

$n$, No. of subjects. Standard error of the mean (SEE) is in ml·kg$^{-1}$·min$^{-1}$.

$C_1$, coefficient describing the minimum walking metabolic rates; $C_2$, coefficient describing speed-dependent increases in metabolic rates; exponent describes speed-dependent increases in metabolic rates.
posed in our title is largely positive. Consequently, we conclude that the energy cost of human walking on firm, level surfaces can be accurately predicted from three basic variables: height, weight, and walking speed.

**Mechanistic Basis of the Height-Weight-Speed Model**

From the outset, we postulated that a direct link between walking gait mechanics and metabolism would explain the stature-related trends long observed for human walking economy. We expected the greater metabolic rates of shorter vs. taller individuals at any given speed, and the greater increases across speed (Fig. 2, A, D, and G) to be fully explained by greater stride frequencies with no appreciable differences in per-stride metabolic costs (ml O₂·kg⁻¹·stride⁻¹). In our model, these expectations translated into per-stride metabolic costs that were theorized to be invariant for subjects of different statures walking at mechanically equivalent speeds. Did these theoretical expectations match the empirical results obtained?

The walking metabolic rates of the four stature groups formed to test hypothesis one, when divided by respective stride rates to obtain per-stride costs, were indeed similar when expressed in relation to our index of equivalent speed (V²·Ht⁻¹, Fig. 3). Both the mean values for the energy expended per kilogram per stride and the sigmoidal patterns of increase across equivalent speeds for the four different stature groups were largely the same, albeit with a consistent trend for the tallest stature group to have marginally greater values than the other three. These results support the basic mechanistic premise of our model that the mass-specific metabolic energy expended to execute each component of the walking stride should be the same at mechanically equivalent speeds for individuals who differ in stature. The coupling observed between gait mechanics and metabolic rates across body sizes at equivalent walking speeds provides two basic conclusions. First, the size-based trends long noted for human locomotor economy across walking speeds and previously considered largely (37, 52, 53) or partially (21, 35) in terms of age are attributable to a common link between body size and gait mechanics. Second, the mass-specific locomotor economy of humans, like that of nonhuman terrestrial species (27, 45, 46), has a per-stride mechanistic basis that becomes evident when comparisons are at mechanically equivalent speeds.

While per-stride metabolic costs can be readily assessed, the mechanistic validity of the three metabolic components of our model (Fig. 1), for reasons both general and specific, is more difficult to evaluate. From a general standpoint, models as parsimonious as the one offered here inevitably simplify biological reality to some degree. From a specific standpoint, evaluating the mechanistic validity of our model’s metabolic components was not an objective that directly aligned with our primary goal of assessing the model’s gross predictive accuracy. These limitations notwithstanding, the contributions of the three metabolic components of the model, and their inferred contributions as a function of absolute and relative walking speeds, did correspond well to theoretical expectation. This is best illustrated by the representative data presented in Fig. 2 in which successive metabolic components were removed to illustrate gross
metabolic rates, net walking metabolic rates, and SDWMRs, respectively, for four individuals who differed in stature. These results support the model’s general validity and suggest the model may have value as an experimental tool for advancing basic understanding. In this latter regard, the parsimony and defined metabolic components of the model should serve as an experimental asset rather than a liability. In contrast to the many empirically derived predictive equations formulated with limited deterministic basis (3, 7, 16, 17, 37, 38, 52), each of the terms in our Height-Weight-Speed model has been defined and, therefore represents a potentially testable biological entity.

**Hypothesis Test One: Body-Size Independence**

As expected for hypothesis test one, the metabolic rates predicted for the same group of reference subjects from model equations derived independently on groups of short, moderately short, moderately tall, and tall subjects differed little in magnitude and not at all statistically. When assessed at the intermediate speeds in our protocol that encompass the speed range humans typically self-select (13, 18, 34), we found the average agreement between the mean values predicted by the four respective equations for the same group of reference subjects was just over 5.0% (Fig. 4A). This led us to conclude that the stature of the subjects on whom the model equation was derived had little effect on the predictions provided. In addition to the predicted means agreeing closely with one another, they also were in good agreement with the values actually measured for the reference subjects at these intermediate speeds, with the average agreement between the means predicted by the four respective equations and those measured being 3.9 ± 3.7%. In contrast, the predictions from five of the leading equations from the literature for the same four speeds (n = 20 comparisons) differed from each other by an average of 13.8 ± 8.4% and from the measured values by an average of 9.5 ± 7.5% (Fig. 4B).

Although our primary test of an effect of body size on the model predictions provided was negative, there was a consistent trend for the tallest subjects to be slightly less economical than strictly theorized. This trend was first apparent in the gross metabolic rate predictions generated by the equation derived on the tallest subjects. These values tended to be slightly greater than those predicted from the equations derived on the three shorter groups, particularly at the slowest walking speeds (Fig. 4A). Mathematically, these small predictive differences resulted from best-fit values for the coefficient \( C_1 \), which describes the increase above RMR constituted by the MWMR, being greatest for the tallest group (Table 2). Indeed, across the four groups, the \( C_1 \) values derived increased slightly, but consistently, with the mean stature of the group. Because these \( C_1 \) differences were small, the resulting differences in metabolic rates predicted were also small, with little effect being discernible in the predictions provided from the equations derived on the shortest three groups. However, in the case of the tallest group, the metabolic rates predicted were slightly, but consistently, greater across walking speeds. While one possible explanation for the limited variation observed in the group \( C_1 \) values derived is modeling artifact, empirical observations indicate otherwise. Specifically, the slightly poorer per-stride economy directly measured for the tallest subject group vs. the other three stature groups across equivalent walking speeds (Fig. 3) indicates that a small, but measurable biological difference is present. Thus, as reflected in the greater \( C_1 \) values derived for the tallest group, our results suggest that human walking economy has a slight stature dependency even after gait mechanics have been accounted for. For reasons not yet clear, tall individuals walk slightly less economically in terms of mass-specific per-stride metabolic costs and related gait mechanics that are incorporated into our model.

**Hypothesis Test Two: Predictive Accuracy**

Fair consideration of the relative predictive accuracy of the two leading literature equations evaluated here warrants the acknowledgment of two factors. First, in contrast to the new model introduced here, the ACSM and Pandolf et al. equations were formulated on, and meant to be applied to, adult-only subject populations. Second, both ACSM and Pandolf et al. are generalized equations that incorporate the influence of factors not presently included in the Height-Weight-Speed model. Specifically, the ACSM equation quantifies the influence of surface inclination in addition to \( M_t \) and speed (3). The Pandolf et al. equation quantifies the influence of surface inclination, load carriage, speed, and terrain (38). Hence, these two leading literature standards were developed for, and have, a greater breadth of application than our Height-Weight-Speed model does at present.

However, a primary motivation for formulating our new model was the recognition that existing predictive models do not include what appears to be one of the three basic determinants of human walking economy: stature. The potential consequences of omitting stature from predictive equations include poorer predictive accuracy and systematic error. These consequences would likely be most evident in the predictions provided for a stature-stratified group of subjects, such as that tested here. However, given the basic influence of stature on gait mechanics and accompanying patterns of locomotor metabolism, predictive accuracy is likely to be compromised among more stature-homogeneous subjects also, but simply to a lesser degree. Both of these expectations were borne out in our results. First, in the original stature-stratified validation group of 39 subjects, the average error of prediction from our Height-Weight-Speed model was less than one-half that of the ACSM and Pandolf et al. equations as hypothesized, being 8.1 ± 6.7% for our model vs. 18.0 ± 13.1 and 21.0 ± 14.4% for ACSM (3) and Pandolf et al. (38), respectively. The greater predictive error in these two established literature equations resulted primarily from consistent underpredictions of the metabolic rates of the shorter subjects in our sample (Fig. 5, A and B). However, when we narrowed the range of statures to include only subjects in the typically adult range of 1.50 to 1.90 m, a roughly twofold difference in predictive accuracy remained. In the latter case, for the 28 subjects in the validation group within this range of typical adult statures, the average error of the individual predictions for the Height-Weight-Speed model was 8.4 ± 7.2 vs. 15.2 ± 10.7 and 17.5 ± 12.5% for ACSM and Pandolf et al., respectively.

Next, we examined the individual-case consequences of including stature, or not, in predictive equations using the walking metabolic rate data acquired from three individual subjects in our data set: one short, one of average height, and
one tall (Fig. 6, A, B, and C, respectively). Also appearing in the three-paneled illustration are the predictions provided for each individual by five well-established literature equations that appear in Table 3: ACSM (3), Pandolf et al. (38), Cotes and Meade (19), van der Walt and Wyndham (51), and Workman and Armstrong (56), as well as the Height-Weight-Speed model introduced here. The first four of the aforementioned predictive equations do not include stature as a predictor and thus provide identical predictions of the mass-specific metabolic rates for each of the three individuals at each of the speeds illustrated. In contrast, the metabolic rates actually measured for these individuals at common speeds vary over a roughly twofold range and do so in inverse relation to their statures. Thus the consequence of not including stature as a predictor is significant underestimations for the short individual and significant overestimations for the tall individual by all four of the predictive equations that do not include stature.

The accuracy of the two remaining equations illustrated, Workman and Armstrong (56) and the Height-Weight-Speed model, is appreciably better for the three individuals illustrated, because these equations do include stature as a predictor. The predictions provided across speed for each of the three individuals by these two equations vary inversely, to some degree, with stature, and therefore in greater accordance with the walking metabolic rates measured. The stature-related variability present is predicted less accurately by the Workman and Armstrong equation, despite its appreciably greater complexity (Table 3), because the quantitative influence of stature was probably not fully discerned during the development of this model. One noteworthy comparison between these latter two equations is that our Height-Weight-Speed model captures a larger proportion of the stature-related variation with an equation that has fewer than half as many terms.

Table 3. Prediction equations from prior literature

<table>
<thead>
<tr>
<th>Equation</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_O_2 (\text{ml} \cdot \text{kg}^{-1} \cdot \text{min}^{-1}) = (0.1-V) + (1.8 \cdot V \cdot G) + 3.5 \cdot M$</td>
<td>ACSM (3)</td>
</tr>
<tr>
<td>$V_O_2 \text{equiv} (W) = 1.5 \cdot M + 2.0 \cdot (M + L)$</td>
<td>Pandolf et al. (38)</td>
</tr>
<tr>
<td>$V_O_2 (l/min) = 0.00800 \cdot M + 0.000245 \cdot M \cdot V^2$</td>
<td>Cotes and Meade (19)</td>
</tr>
<tr>
<td>$V_O_2 (l/min) = 0.00599 \cdot M + 0.000245 \cdot M \cdot V^2$</td>
<td>van der Walt and Wyndham (51)</td>
</tr>
<tr>
<td>$V_O_2 (l/min) = Ht \cdot (0.0136 \cdot Ht - 0.375)^{-1} \cdot (1.92 \cdot V + 1.766 \cdot 1.445 \cdot M - 0.82 \cdot V^2 - 3.94 \cdot V + 9.66) \cdot 10^{-5}$</td>
<td>Workman and Armstrong (56)</td>
</tr>
</tbody>
</table>

$V$, velocity; $M$, body mass; $Ht$, height; $L$, load (body mass units); $G$, grade (%), $\eta$, terrain factor (arbitrary units); $V_O_2 \text{equiv}$, equivalent $V_O_2$: in Watts (W).

Table 4. Schofield et al. equations for predicting RMR from sex, age, and body mass

<table>
<thead>
<tr>
<th>Age, yr</th>
<th>Male</th>
<th>Female</th>
</tr>
</thead>
<tbody>
<tr>
<td>Under 3</td>
<td>$\text{RMR} = 8.603 - (4.388 \times \text{wt}^{-1})$</td>
<td>$\text{RMR} = 8.43 - (4.491 \times \text{wt}^{-1})$</td>
</tr>
<tr>
<td>3–10</td>
<td>$\text{RMR} = 3.282 + (72.899 \times \text{wt}^{-1})$</td>
<td>$\text{RMR} = 2.937 + (70.239 \times \text{wt}^{-1})$</td>
</tr>
<tr>
<td>10–18</td>
<td>$\text{RMR} = 2.557 + (95.149 \times \text{wt}^{-1})$</td>
<td>$\text{RMR} = 1.935 + (100.124 \times \text{wt}^{-1})$</td>
</tr>
<tr>
<td>18–30</td>
<td>$\text{RMR} = 2.177 + (100.055 \times \text{wt}^{-1})$</td>
<td>$\text{RMR} = 2.142 + (70.343 \times \text{wt}^{-1})$</td>
</tr>
<tr>
<td>30–60</td>
<td>$\text{RMR} = 1.658 + (126.209 \times \text{wt}^{-1})$</td>
<td>$\text{RMR} = 1.175 + (122.236 \times \text{wt}^{-1})$</td>
</tr>
<tr>
<td>Over 60</td>
<td>$\text{RMR} = 1.693 + (84.957 \times \text{wt}^{-1})$</td>
<td>$\text{RMR} = 1.313 + (95.184 \times \text{wt}^{-1})$</td>
</tr>
</tbody>
</table>

Resting metabolic rate (RMR) is in $O_2$ uptake units (ml$\cdot$kg$^{-1}\cdot$min$^{-1}$). Body weight (wt) is in kg.
Concluding Remarks

The primary goal of our basic approach to integrating body size, gait mechanics, and locomotor metabolism was to provide a translational advance in the form of an improved generalized equation for predicting the energy requirements of human walking on firm, level surfaces. Although gait mechanics and physiological function during walking are complex, our basic approach led to an equation with one speed-dependent term and two constants to explain the variation in walking metabolism introduced by both body size and walking speed. In closing, we put forth several suggestions for easy utilization of the translational tool offered. First, we recommend using the sex, age, and mass-based RMR estimates provided by the Schofield et al. equations in Table 4, given their well-established accuracy and the impracticality of obtaining direct RMR measurements in most circumstances. Second, because the second metabolic term in the model is a multiple of the first, the two can be combined into a single term. Since the model value empirically identified for the second metabolic term was generally equal to the first (Table 2, see C1 values ≈ 1.0), this combined term can be well described as the RMR multiplied by two. Third, although the model exponent that originally provided the best fit to our experimental group data was 2.34, the predictive improvement offered vs. the theorized exponential value of 2.00 was negligible. In combination, these observations lead us to offer the following simplified equation for general predictive purposes:

\[ \dot{V_{O_2}}_{\text{gross}} = 2.0 \cdot \dot{V_{O_2}}_{\text{rest}} + 5.6 \cdot V^{2.0} \cdot H^{-1} \]  

(2)

where \( \dot{V_{O_2}}_{\text{gross}} \) and \( \dot{V_{O_2}}_{\text{rest}} \) (Ref. 44, Table 4) are expressed in ml O2·kg\(^{-1}\)·min\(^{-1}\), \( V \) is in m/s, and \( H \) is in meters. The accuracy of prediction using the simplified expression of the model in Eq. 2 differs negligibly from the original. An SEE increase of <0.2 ml O2·kg\(^{-1}\)·min\(^{-1}\) resulted from using the simplified rather than the originally derived equation to predict the metabolic rates of the validation group subjects.

Clearly, additional experimental work is required to determine how well the Height-Speed model might describe walking metabolism under conditions not tested here, such as graded surfaces, load carriage, and variable terrain. Nonetheless, by including the influence of stature on gait mechanics and walking metabolism as we have, our new model provides accurate predictions of the energy requirements of level walking and does so from a concise equation with discrete, testable components.

ACKNOWLEDGMENTS

We thank Ann Adolph and Firoz Vohra for technical support, Mercedes Alejandro for recruitment and scheduling assistance, and our 78 subjects for their willing efforts. Rosalind Sandell and Ken Clark contributed substantially to data analysis and manuscript preparation. Laurence Ryan provided valuable guidance on the derivation of the model and helpful input throughout. Dr. Kyle Roberts assisted with the statistical analysis.

GRANTS

This work was made possible in part from grant DAMD17-03-2-005 from the Medical and Materiel command of the United States DOD and internal funds from Southern Methodist University to P. G. Weyand.

DISCLOSURES

No conflicts of interest, financial or otherwise, are declared by the author(s).

AUTHOR CONTRIBUTIONS


REFERENCES


