Flow-induced shear strain in intima of porcine coronary arteries

Wei Zhang, Yi Liu, and Ghassan S. Kassab

Departments of Biomedical Engineering, Surgery, and Cellular and Integrative Physiology, Indiana University Purdue University Indianapolis, Indianapolis, Indianapolis

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Zhang W, Liu Y, Kassab GS. Flow-induced shear strain in intima of porcine coronary arteries. J Appl Physiol 103: 587–593, 2007. First published May 24, 2007; doi:10.1152/japplphysiol.00199.2007.—The in vivo circumferential strain has a small variation throughout the vascular system (aorta to arterioles). The axial strain has also been shown to be nearly the same as the circumferential strain under physiological loading. Since the endothelium is mechanically much softer than the media-adventitia in healthy arteries, the porcine intima was considered as a mechanically distinct layer from the media-adventitia in a two-layer computational model. Based on the simulation result, we hypothesize that the flow-induced shear strain in intima can be of similar value as the pressure-induced circumferential strain in healthy coronary arteries, even though the shear stress is orders of magnitude smaller than the circumferential stress. The nearly isotropic deformation (circumferential, axial, and shear strains) may have important implications for mechanical homeostasis of endothelial cells, mechanotransduction, growth, and remodeling of blood vessels.

BLOOD VESSELS ARE SUBJECTED to mechanical loadings, such as internal blood pressure, flow-induced shear, and tethering from surrounding tissues. It is believed that a mechanical homeostatic state exists in blood vessels (22). Although stress and strain may vary throughout the circulatory system, the variation is relatively small, particularly for the strain (12). A remodeling response is initiated when a change in the loading conditions (blood pressure, flow, surrounding tissue, etc.) perturbs the mechanical homeostasis. The response is an adaptation to restore the homeostatic state (22). The mechanical homeostasis can be described by the uniform strain hypothesis, which states that the in vivo transmural circumferential strain is uniform in the vessel wall (13, 37). This hypothesis can be extended for the mean wall strain measured in reference to the zero-stress state, which has been found to be remarkably uniform in the coronary arterial tree and aorta (12). Zhang et al. (42) found that the circumferential and axial strains tend to be uniform and equal in the arterial wall under physiological loading.

An arterial wall is typically composed of three functional layers from inner to outer wall surfaces: intima, media, and adventitia. The intima comprises a single layer of endothelial cells and a basal lamina in normal vessels. The media consists mainly of smooth muscle cells, elastin, and collagen fibrils. The adventitia contains mainly collagen fibers, ground substance, fibroblasts, and fibrocytes. Since the intima is very thin and much more compliant than the media and the adventitia, its material property is generally ignored. Consequently, blood vessels are typically viewed either as a single layer of homogeneous material (3, 5, 19, 32) or as a two-layer composite of intima-media and adventitia under normal conditions (16, 26, 27, 34, 39). It has been well accepted that endothelial cells (the major constituent of intima) are very sensitive to changes in flow-induced shear stress, and many biological activities are related to this shear loading (1, 4, 7, 30, 31, 35, 38, 40). For instance, the diameter and thickness of the vessel wall are modulated by flow-induced shear stress, even though the shear stress is several orders of magnitude smaller than the circumferential wall stress. This raises the question of why blood vessels are so sensitive to such small loading. Our hypothesis is that shear strain can be substantial in endothelial cells, despite the small shear stress. Our rationale is that the deformation of endothelium can be underestimated if the mechanical property of intima is not considered individually.

Most previous investigations on the mechanics of vessel wall have focused on the pressure-induced normal stress and strain. The shear strain, however, due to the relatively small magnitude of flow-induced shear stress, has not received significant attention. In this study, we will examine the shear deformation in arterial walls under physiological pressure and flow conditions. We will demonstrate that the flow-induced shear strain in the intima layer can be of the same magnitude as normal (circumferential and axial) strains caused by pressure and axial prestretch. This result may shed light on mechanotransductive mechanisms in blood vessels, as well as arterial growth and remodeling modulated by the mechanical environment of endothelial cells.

Glossary

- \( r \) Radial coordinate, deformed
- \( \theta \) Circumferential coordinate, deformed
- \( z \) Axial coordinate, deformed
- \( w \) Axial displacement of arterial wall
- \( R \) Radial coordinate, zero stress
- \( \Theta \) Circumferential coordinate, zero stress
- \( Z \) Axial coordinate, zero stress
- \( \Phi \) Opening angle
- \( \chi \) Defined as \( \chi = \pi/(\pi - \Phi) \)
- \( H \) Hydrostatic stress
- \( P \) Internal blood pressure
- \( \tau \) Shear stress at inner surface
- \( \mu \) Coefficient of viscosity
- \( Q \) Rate of flow
- \( L \) Length for which pressure drops \( \Delta P \)
- \( W \) Strain energy function
- \( C \) Parameter in Fung strain energy (kPa)

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Materials and Methods

Mathematical model. To make the problem analytically tractable, we consider an artery segment that is mechanically homogeneous, cylindrically orthotropic, and volumetrically incompressible in each respective layer. It is further assumed that the artery remains as a straight cylinder with uniform thickness along the axial direction in the loaded state (Fig. 1A). The zero-stress state is represented by an open sector as shown in Fig. 1B, wherein the opening angle \( \Phi \) characterizes the circumferential residual strain (5) at the no-load state (i.e., a tube with zero pressure and shear). The deformed cross section is schematically shown in Fig. 1C, where the axial \( (z) \) direction is pointing out of the \( r = 0 \) plane. We employ a two-layer model in which the intima is considered different from the rest of the wall; i.e., the media-adventitia layer (Fig. 1D). This treatment differs from the classic intima-media and adventitia two-layer model (16, 39), since the significance of intima layer is of interest in this work.

More specifically, we assume that the artery deforms uniformly in the axial direction and the deformation is axisymmetric, so that all variables depend only on the radial coordinate \( r \) and the shear stress in the \( r = 0 \) plane is zero throughout the arterial wall. Thus the nontrivial equations of equilibrium on the deformed configuration are simplified as:

\[
\frac{d\sigma_r(r)}{dr} + \frac{\sigma_r(r) - \sigma_{\theta\theta}(r)}{r} = 0
\]

where \( \sigma_{jk} \) \( (j, k = r, \theta, z) \) denotes Cauchy stress. Using the approaches developed by Humphrey and Na (19) and Rachev et al. (32) while considering the boundary condition \( \sigma_r(r) = -P \) (where \( P \) is blood pressure), the following expressions are obtained (see Appendix for details):

\[
\sigma_{\theta\theta}(r) = \int_0^r \left( \lambda_\theta \frac{\partial W}{\partial \epsilon_{\theta\theta}} - \lambda_\phi \frac{\partial W}{\partial \epsilon_{\phi\phi}} \right) \frac{dr}{r} - P
\]

\[
\sigma_{\phi\phi}(r) = \sigma_r(r) + \lambda_\phi \frac{\partial W}{\partial \epsilon_{\phi\phi}} - \lambda_\theta \frac{\partial W}{\partial \epsilon_{\theta\theta}}
\]

\[
\sigma_r(r) = \sigma_r(r) + \lambda_\theta \frac{\partial W}{\partial \epsilon_{\theta\theta}} + \lambda_\phi \frac{\partial W}{\partial \epsilon_{\phi\phi}} + 2\lambda \xi_{\phi} \frac{\partial W}{\partial \epsilon_{zr}}
\]

where \( W(E_{jk}) \) is the strain energy, \( E_{jk} \) \( (j, k = r, \theta, z) \) is the Green strain (see Eq. A4 in Appendix), \( \lambda_{\alpha} \) \( (m = r, \theta, z) \) are stretch ratios, and \( \xi_\alpha \) are defined as \( d\alpha/dr \) with \( w \) being the displacement that induces shear strain. It is noted that \( W \) is different for the arterial layers, and hence two sets of parameters are needed for the intima and media-adventitia, respectively.

If we consider the boundary condition \( \sigma_r(r) = -\tau \) (where \( \tau \) is the blood shear stress at the inner surface, see Fig. 1A), the solution of Eq. 2 can be obtained as \( \sigma_r(r) = -\tau r / r \), which implies that the perivascular tethering tissue must provide a shear force \( -\tau r / r \) on the outer surface to balance the flow-induced shear load. The shear stress at given radial coordinate \( r \) can be used to compute \( \xi_\alpha \) from Eq. A11:

\[
\lambda_\theta \frac{\partial W}{\partial \epsilon_{\theta\theta}} + \lambda_\phi \frac{\partial W}{\partial \epsilon_{\phi\phi}} = -\tau r / r
\]

Fung’s exponential strain energy function is used, which takes the following form (5, 19):

\[
W = \frac{C}{2} [\exp(b_r E_{11}^2 + b_r E_{22}^2 + b_r E_{33}^2 + 2(b_r E_{12} E_{21} + b_r E_{13} E_{31} + b_r E_{23} E_{32})) - 1]
\]

The constants \( C \) and \( b_1-b_7 \) are material parameters determined from experiment.

Simulation parameters. The dimensions of the artery at zero-stress state were taken as \( R_i = 12.78 \) mm, \( R_o = 13.12 \) mm, \( \Phi = 163^\circ \) (Kassab et al., unpublished data corresponding to Ref. 39). The intima (2 ~ 5 \( \mu m \) thick) was assumed to occupy 2% of the wall thickness (100 ~ 250 \( \mu m \)). The axial stretch ratio was chosen as \( \lambda_z = 1.4 \) (27). The Fung-model parameters \( C \) and \( b_1-b_7 \) for the intima-porocine left anterior descending (LAD) artery are listed in Table 1, as previously reported by our group (39). The porcine coronary artery elastic modulus was estimated to be 170 kPa at physiological loading based on experimental data (26). According to Sato et al. (36), the Young’s modulus of porcine aortic endothelial cells is on the order of 0.1 kPa. Hence the modulus in endothelium is \( \sim 1,000 \) times smaller than the intact artery. Since complete model parameters are not available for intima, the parameter \( C \) was taken to be 1% and \( b_\alpha \) \( (\alpha = 1, \ldots, 7) \) to be 10% of those of the intact porcine LAD artery reported by Wang et al. (39). To maintain the overall elastic stiffness of the artery at \( P = 100 \) mmHg, parameter \( C \) in the media-adventitia was increased by 1.65% from the value in intact wall (Table 1). Note that parameter \( C \) in the Fung model is a scale factor, and parameter \( b_\alpha \) reflects the material nonlinearity (curvature of stress-strain relation). Both \( C \) and \( b_\alpha \) have been rescaled to make the elastic modulus (proportional to \( C b_\alpha \)) of intima 1,000 times smaller. Sato et al. (36) considered a linear viscoelastic model for endothelial cells. Here we have chosen small \( b_\alpha \).
Table 1. Fung model material properties for porcine left anterior descending coronary artery

<table>
<thead>
<tr>
<th></th>
<th>C, kPa</th>
<th>b1</th>
<th>b2</th>
<th>b3</th>
<th>b4</th>
<th>b5</th>
<th>b6</th>
<th>b7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intima</td>
<td>8.76</td>
<td>1.21</td>
<td>3.39</td>
<td>0.79</td>
<td>0.35</td>
<td>0.11</td>
<td>0.10</td>
<td>2.89</td>
</tr>
<tr>
<td>Media-adventitia</td>
<td>8.90</td>
<td>1.21</td>
<td>3.39</td>
<td>0.79</td>
<td>0.35</td>
<td>0.11</td>
<td>0.10</td>
<td>2.89</td>
</tr>
</tbody>
</table>

C and b parameters in Fung strain energy.

(Table 1) to ensure nearly linear stress-strain relationship of intima. More details about parameter selection are provided in the APPENDIX.

The equations were solved numerically with the finite difference method, as outlined in Fig. 2. The solution was adopted when the difference between the radial stress at the outer surface [σr(r_o)] in Eq. 3] and the prescribed pressure boundary condition is <10^-8 kPa. A simple test was done to verify that the unloaded state [P = 0, σr(r_o) = 0, and λz = 1.0] can be achieved with the above parameters (the resulting no-load r_1 = 1.06 mm and r_0 = 1.39 mm), for which the residual stresses and strains are distributed across the arterial wall in a pattern similar to those shown in Ref. 5; e.g., circumferential components are compressive at the inner surface and tensile at the outer surface.

For the physiological state, we considered σr(r_1) = -13.33 kPa (the average value of systole and diastole pressure, P = 100 mmHg), while the constraint at the outer surface was taken to be σr(r_o) = -8.67 kPa (65 mmHg), which stems from the tethering myocardium (14). The choice of the external pressure was based on the consideration of myocardial constraint and the resulting outer radius of the vessel (1.81 mm at loaded state), consistent with the reported pressure-diameter relation of in situ porcine LAD artery (14).

RESULTS

Although only one numerical case is presented here, we found that similar results can be obtained in simulations with typical material properties. Hence the present results have generality.

Under simultaneous internal pressure P = 100 mmHg, external pressure 65 mmHg, and blood shear stress τ = 1.5 Pa (15 dyn/cm²), it is found that the shear strain is significant in the intima (Er_intima ≈ 0.08), but close to zero in media-adventitia (Fig. 3A). The shear strain increases sharply at the interface of intima and media-adventitia (at normalized wall thickness of 2%, Fig. 3C), so does the radial strain (slightly) due to the contribution of Er in E_r (i.e., discontinuous Er results in discontinuous E_r, Eq. A4). The radial stress is compressive in the whole vessel wall, but the circumferential and axial stresses change abruptly from compressive to tensile across the interface of intima and media-adventitia layer (Fig. 3B). All normal stresses are nearly equal to the negative blood pressure (13.33 kPa) in the intima layer (Fig. 3D), indicating that the intima is under a hydrostatic state approximately. The shear stress is always small, however, as expected (see right-hand side of Eq. 6).

For comparison with the classic two-layer (intima-media and adventitia) model, we chose C = 5.11 kPa, b_1 = 2.47, b_2 = 3.09, b_3 = 0.95, b_4 = 0.45, b_5 = 0.06, b_6 = 0.10 for the intima-media layer, and C = 9.05 kPa, b_1 = 0.62, b_2 = 2.27, b_3 = 1.67, b_4 = 0.34, b_5 = 0.11, b_6 = 0.07 for the adventitia layer (39). The b_2 = 2.89 was assumed for both layers, and the intima-media was taken to be 60% of the total wall thickness. Under the same boundary conditions stated above, the (negative) maximum shear strain was found to be 3.8 x 10^-5 at the outer surface of the intima-media. Thus the large flow-induced shear strain in intima is overshadowed in the intima-media and adventitia model.

For the same material properties in Table 1, the circumferential and axial strains do not change noticeably from Fig. 3A when the applied shear stress τ is increased to 9 Pa. However, the magnitude of shear strain in intima increases almost linearly with τ, and it approaches the magnitude of normal strains, e.g., | Er_intima | ≈ 0.4 at τ = 9 Pa (Fig. 4A). The radial strain in media-adventitia is also similar to Fig. 3A, whereas its magnitude decreases nonlinearly with τ in intima (Fig. 4A), because the increase of the term Er in Eq. A4 leads to increasing Er (its magnitude decreases since E_r is negative). This result implies that shear and radial strains in endothelial cells strongly depend on the flow conditions (normal flow vs. flow overload).

If the elastic stiffness in intima is smaller than the assumed value, the magnitude of shear strain will increase further and become comparable with the normal strains, even under normal flow (τ ~ 1.5 Pa). Figure 4B shows that the magnitude of shear strain on the inner surface increases (but radial strain decreases) nonlinearly with decreasing b_7 for intima (other parameters were kept unchanged). At b_7 = 0.05, the magnitude of shear strain (| Er_intima | = 0.44) approaches that of the circumferential and axial strains. The shear stress, on the other hand, remains several orders of magnitude smaller than the normal stresses, even in flow overload.
DISCUSSION

Our findings show that flow-induced shear strain in intima (largely composed of endothelium) may reach the magnitude of normal strains caused by blood pressure and axial stretch, despite the relatively small shear load. This may have interesting implications, as discussed below.

Strain or stress as mechanical stimulus. Experiments by Lu and Kassab (25) revealed that the forward flow promotes nitric oxide production in arterial wall under normal flow, but significantly reduces nitric oxide concentration under reversed flow. This phenomenon has been attributed to increased production of superoxide anion in reversed flow. The biological response of endothelial cells to the same magnitude of wall shear stress (but opposite in direction) is very different. It may be that the shear strain on the endothelial cells is very different, depending on the direction, as speculated below.

The residual strain in blood vessels is closely related to the remodeling process (10). There is enormous evidence that circumferential residual strain (characterized by the opening angle) in arteries changes with remodeling, e.g., Liu and Fung (24). Davies (7) pointed out that prolonged steady flow results in reorganization of endothelial surface and significant reduction of peak shear stress compared with no-flow conditions, which implies that a residual shear strain may exist in the arterial wall due to remodeling under chronic flow-induced shear. As illustrated in Fig. 5, if residual shear strain exists (left), the shear deformation might be quite different for the same shear stress under forward and reversed flows (middle and right). The flow-dependent reorientation of endothelial cells (7, 30) could be related to the residual shear strain. Nonetheless, three-dimensional (3D) reconstructions of endothelial cells under various shear loadings are needed to confirm this hypothesis.

It has been well documented that stretch (or pressure) and shear (frictional flow force) are mechanical stimuli that influence the activity of endothelial and vascular smooth muscle cells (15, 23). The flow reversal effect (25) supports the implication that strain may be the mechanical stimulus, because the biological response for the same shear stress (forward vs. reversed) is very different. It is also possible that the asymmetric response of endothelium to directionality may be due to different elastic properties in the forward and reverse directions. It is known that actin microfilaments (as well as other cytoskeletal elements) act very differently under tension and compression. This alternative hypothesis deserves further attention. In reality, both endothelial residual strain and differences in tension and compression of cytoskeleton may be implicated.

Rachev (33), Fridez et al. (8), and Gleason and Humphrey (11) have studied the remodeling of carotid artery, including passive and active behaviors, by considering blood flow and pressure loading. The potential residual shear strain may be included in such analyses, as it could play an important role in interpreting flow-related remodeling and mechanotransduction.

Critique of method. Theoretically, if material property does not change along the axial direction, the arterial diameter will decrease along the flow direction. This is because flow is a result of pressure gradient. Using Poiseuille’s law (10), the internal shear stress can be approximated by

$$\tau = \frac{4\mu Q}{\pi r^4} = \frac{r\Delta P}{2L}$$

(8)

where \(\mu\) denotes the coefficient of viscosity, \(Q\) is the rate of flow, and \(L\) equals the length along axis \(z\) when pressure drop is \(\Delta P\). Therefore, all of the variables depend on coordinate \(z\) as well as on \(r\) if flow exists. Nevertheless, this dependence on \(z\)
though we selected the parameter obtained from the shear loading endothelial experiments. As arterial wall remodeling is a secondary effect in a typical blood vessel segment of interest \((L \sim 1 \text{ cm})\), because the pressure drop is small and the change of lumen radius is insignificant. Thus our assumption that all variables are independent of \(z\) is reasonable.

Since the 3D mechanical data for the endothelium do not exist, we assumed a Fung-type exponential energy function, which is known to characterize many biological tissues \((10)\), and estimated the respective material parameters. The one-dimensional (1D) modulus for the endothelium \((36)\) was compared with that of vessel wall \((39)\) and found to be 1,000 times softer. The issue was to transform the 1D experimental data reported for the endothelium to a 3D constitutive relation. The material parameter selection for the estimated 3D endothelial constitutive relation preserves both the linearity (determined by \(b_a\)) and the magnitude of the modulus (determined by \(C\)) obtained from the shear loading endothelial experiments. Although we selected the parameter \(C = 1\%\) and \(b_a (\alpha = 1, \ldots, 7)\) as 10% of those of the intact artery, \(C\) of 10% and \(b_a\) of 1% would lead to the same conclusion. Similarly, other combinations of \(C\) and \(b_a\) lead to a similar conclusion regarding the shear deformation, as long as we maintain the observed linearity for the endothelium (decrease \(b_a\)) and modulus \((Cb_a\) of 0.1%).

Here the media and adventitia have been regarded as one layer because their mechanical properties are on the same order. The conclusion that shear strain in intima may be significant will not change if media and adventitia layers are treated differently. We assumed that the intima is equal to 2% of the wall thickness, which is reasonable in young and healthy blood vessels. In diseased or aged arteries, the intima becomes thicker and stiffer (17), and its role becomes more important in tensile support (28, 29). The thickened and hardened intima will change the strain distribution, which may affect mechanotransduction, if strain is the stimulus for homeostasis.

The arterial wall has been modeled as an elastic material, and hence the stress and strain distributions are independent of the loading path. If the viscoelastic behavior of blood vessels is considered, the loading sequence must be taken into account. A 3D finite-element model for the viscoelastic deformation of endothelial cells has been considered in the literature \((21)\) and is beyond the scope of the present study. Other issues, such as mechanical property of perivascular tethering, non-Newtonian flow, complex geometry, and heterogeneity of the vessel wall, will make analytical modeling impractical. Some of these effects can be implemented in finite-element method considering fluid-solid interaction, constraint of surrounding tissues, etc. \((20, 41)\).

It should be noted that our modeling has been conducted on the macroscopic level based on continuum mechanics. Therefore, stress and strain in the arterial wall must be interpreted as the average values of various microstructural constituents. This is different from the works of Huang et al. \((18)\) and Karcher et al. \((21)\), as well as other investigations, where details of stresses and strains in the individual cells with realistic shapes and geometries have been considered. Despite the lack of microstructural details, the present model captures the essence of the question raised.

Chaudhry et al. \((3)\) reported that residual stress and strain reduce the circumferential stress gradient in oscillating arterial wall. Cinthio et al. \((6)\) studied the longitudinal movement of arterial wall in cardiac cycle and concluded that the shear strain in artery may be substantial. Based on a general formulation, including dynamics and smooth muscle active terms developed by Humphrey and Na \((19)\), the current model can be extended to investigate arterial behavior involving pulsatile blood flow, smooth muscle cell activity, and other physiological features.

**Summary and significance of study.** A computational model that takes into account coupled pressure and shear loading was used to analyze stress and strain distributions in the porcine coronary artery. It is found that the flow-induced shear strain in intima can be as large as normal strains caused by internal blood pressure. This finding may have significant implications for interpreting vascular behaviors, such as vasoactivity due to change of blood pressure and flow, arterial wall remodeling, and pathophysiology (atherosclerosis). It may shed light on the mechanical stimulation mechanisms in the endothelial cell.

It is suggested that the intima layer should be considered separately to investigate the role of flow-induced shear, since the deformation in the intima may be overshadowed by the

![Fig. 4. Variations of shear strain and radial strain magnitudes on the inner arterial wall when applied shear stress \(\tau\) is increased (A) and parameter \(b_7\) in intima is decreased (B).](http://jap.physiology.org/)

![Fig. 5. Schematic drawing of the arterial wall at zero-stress state with a released residual shear strain (left), under normal flow (middle), and reverse flow (right). The horizontal arrows indicate the shear stress directions.](http://jap.physiology.org/)
media and adventitia layers, which have much larger elastic moduli (flow-induced shear strain was found to be $E_{rr} \approx -3 \times 10^{-5}$ in a one-layer model with intact properties listed in Table 1, or a classic intima-media and adventitia two-layer model as mentioned earlier). From the numerical results of the intima and media-adventitia two-layer model, we hypothesize that shear strain may be a stimulus for the endothelium. In this regard, strain may be a good measure to correlate with biochemical response of blood vessels. This hypothesis needs to be explored in future experimental studies.

**APPENDIX**

*Deformation and stress.* As illustrated in Fig. 1, the coordinates in the deformed configuration ($r, \theta, z$) are related to those in the zero-stress state ($R, \Theta, Z$) by

$$r = r(R), \quad \theta = \chi(\Theta), \quad z = \lambda Z + w(R)$$

where $\chi = \pi/(\pi - \Phi), \lambda$, is the stretch ratio in the axial direction, and $w$ is the relative displacement in the $z$ direction. The deformation gradient matrix with respect to the zero-stress state is

$$[F] = \begin{bmatrix} \frac{\partial r}{\partial R} & \frac{\partial r}{\partial \Theta} & \frac{\partial r}{\partial Z} \\ \frac{\partial R}{\partial \Theta} & \frac{\partial \Theta}{\partial \Theta} & \frac{\partial Z}{\partial \Theta} \\ \frac{\partial R}{\partial Z} & \frac{\partial \Theta}{\partial Z} & \frac{\partial Z}{\partial Z} \end{bmatrix} = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ \xi_w & 0 & \lambda_z \end{bmatrix}$$

(Eq. A2)

where $j, k = r, \theta, z$, and

$$\lambda_j = \frac{dr}{dR}, \quad \lambda_\theta = \frac{\chi r}{R}, \quad \xi_w = \frac{dw}{dR}$$

The corresponding Green strain tensor $E_{jk} = \frac{1}{2} (F_{ij} F_{jk} - \delta_{jk})$, where $l = r, \theta, z$ denotes the dummy index for Einstein summation, is

$$[E] = \begin{bmatrix} \lambda^2 + \xi_w^2 - 1 & 0 & \lambda \xi_w \\ 0 & \lambda^2 - 1 & 0 \\ \lambda \xi_w & 0 & \lambda_z^2 - 1 \end{bmatrix}$$

(Eq. A4)

If we assume incompressibility [det($F$) = 1 in Eq. A2], we have

$$\lambda_z = \frac{R}{\chi \lambda_r}$$

(Eq. A5)

and

$$r = \sqrt{r^2 + \frac{R^2 - R_z^2}{\chi \lambda_r}}$$

(Eq. A6)

Cauchy stress tensor is computed as (19)

$$\sigma_{jk} = F_{ij} F_{km} \frac{\partial W}{\partial E_{mn}} + H \delta_{jk}$$

(Eq. A7)

where $H = H(r)$ is the hydrostatic stress due to incompressibility. Specially, the nonzero stress components are

$$\sigma_{rr} = \lambda_z^2 \frac{\partial W}{\partial E_{rr}} + H$$

(Eq. A8)

$$\sigma_{\theta\theta} = \lambda_z^2 \frac{\partial W}{\partial E_{\theta\theta}} + H$$

(Eq. A9)

$$\sigma_{zz} = \lambda_z^2 \frac{\partial W}{\partial E_{zz}} + \xi_w^2 \frac{\partial W}{\partial E_{zz}} + 2\lambda \xi_w \frac{\partial W}{\partial E_{zr}} + H$$

(Eq. A10)

$$\sigma_{rz} = \lambda_z \lambda_r \frac{\partial W}{\partial E_{rz}} + \lambda \xi_w \frac{\partial W}{\partial E_{rz}}$$

(Eq. A11)

Equations 3–6 can be derived from Eqs. 1, 2, and A8–A11 when considering boundary conditions in the radial direction.

**Material parameters for Fung model.** The parameters in the normal directions for an intact vessel were taken as the average values reported by Wang et al. (39). For the parameter $b_7$, we assumed that the shear modulus in the $rz$ plane is similar to that in the $\theta z$ plane. The shear modulus 160 kPa in the $\theta z$ plane (27) for $\lambda_z = 1.4$, and circumferential stress 90 kPa ($\lambda_z = 1.6$) were used to estimate $b_7 = 2.89$ for Fung model.

The mechanical behavior of endothelium, which is significantly different when the cells are viewed as fluidlike or solidlike (9), is essential to predict the correct stress and strain in intima. Caille et al. (2) adopted an isotropic material model (Mooney–Rivlin law) to fit the experimental force-deformation curves of bovine aortic endothelial cells and determined that their elastic moduli are on the order of 0.5 kPa for spread and round cells and 5.0 kPa for cells with nuclei. Sato et al. (36) reported the linear viscoelastic properties of cultured endothelial cells under shear stress. They estimated that Young’s modulus of porcine aortic endothelial cells is on the order of 0.1 kPa. Considering that the porcine coronary artery elastic modulus is ~170 kPa (26), it is rationalized that the elastic stiffness of porcine endothelium is ~1,000 times smaller than that of the overall vessel. Due to the lack of data, we assumed that the Fung model applies to the intima layer. The properties for intima (basal lamina and subendothelial layer are assumed that the Fung model applies to the intima layer. The properties for intima (basal lamina and subendothelial layer are assumed to have the same property as endothelium), as given in Table 1, were selected to ensure this modulus criterion (modulus is proportional to the product of $C$ and $b_7$).

It should be noted that shear modulus in the $rz$ plane may be different from that in the $\theta z$ plane. In qualitative modeling, accurate experimental results are desired. Since the endothelial cells are soft and thin, it is difficult to obtain their 3D constitutive equation and material parameters. Hence, direct measurement of stress-strain relationship of intima remains a technical challenge.

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**REFERENCES**


